

Don **MATHEMATICS**

- ❖ POINTS TO REMEMBER
- ❖ TEXTUAL QUESTIONS SOLVED
- ❖ ENORMOUS CREATIVE QUESTIONS
- ❖ UNIT QUESTION PAPERS
- ❖ MODEL QUESTION PAPER

8

UPDATED EDITION



Don Publications (P) Ltd.

Don

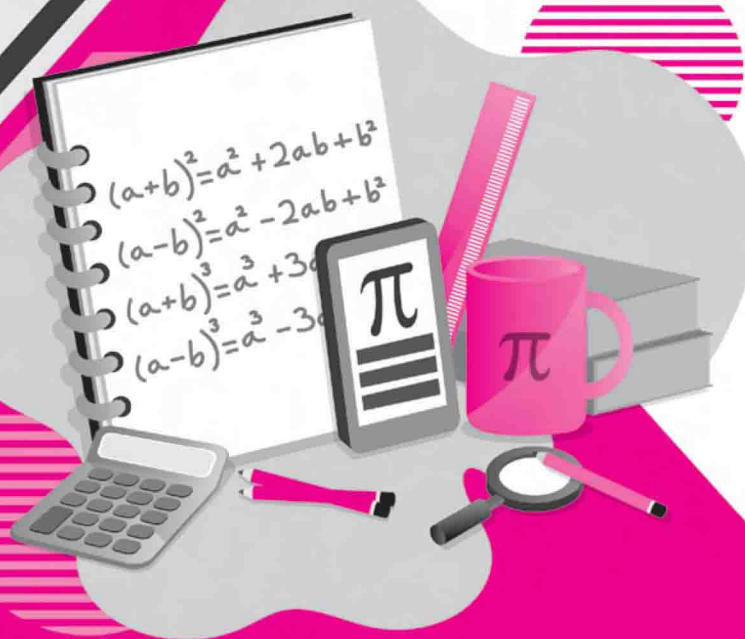
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2023 - 2024

FULLY REVISED EDITION

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8th Standard

MATHEMATICS

(As per the latest syllabus)

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PREFACE

Don Mathematics for 8th standard is prepared based on the Tamilnadu Government latest text book and new syllabus.

In this book, all the textual questions are solved. Also enormous additional CREATIVE questions are given with full solution. And Unit Question Papers are given for all the chapters. A Model Question Paper is also given at the end of this book.

We firmly believe that this book will be of immense help to the students to score centum in their exam.

Wishing you all the best!

S.A. Suresh Kumar MCA, MBA
for **Don Publications (P) Ltd.**

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1

NUMBERS

POINTS TO REMEMBER

- A rational number can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$
- All natural numbers, whole numbers, integers and fractions are called rational numbers.
- Every rational number can be represented on a number line.
- 0 is neither a positive nor a negative rational number.
- A rational number $\frac{a}{b}$ is said to be in the standard form if its denominator b is a positive integer and HCF of $(a, b) = 1$.
- There are unlimited numbers of rational numbers between two rational numbers.
- $\frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$, Where $b \neq 0$
- The average of two numbers a and b is $\frac{1}{2}(a+b)$
- For any non-zero a, b, c and d , we have

i) $\left(\frac{a}{b}\right) \div c = \frac{a}{bc}$	ii) $a \div \left(\frac{b}{c}\right) = \frac{ac}{b}$	iii) $\frac{a}{b} \div \left(\frac{c}{d}\right) = \frac{ad}{bc}$
iv) $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	v) $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$	vi) $\frac{a}{b} + \frac{c}{d} \neq \frac{a+c}{b+d}$

Properties of rational numbers:

- **Closure property:**
For any two rational numbers a and b , $a + b$ and $a \times b$ are unique rational numbers.
- **Commutative property:**
For any two rational numbers a and b , $a + b = b + a$ and $a \times b = b \times a$
- **Associative property:**
For any three rational numbers a, b, c
 $(a + b) + c = a + (b + c)$; $(a \times b) \times c = a \times (b \times c)$
- **Additive and Multiplicative identity property:**
For any rational numbers 'a' there exists unique identity elements 0 and 1 such that (i) $0 + a = a$
 (ii) $1 \times a = a$

➤ **Additive and Multiplicative inverse property:**

For any rational number 'a' there exists unique rational number $-a$ such that

$$a + (-a) = 0 = (-a) + a$$

For any non-zero rational number 'b' there exists a unique rational number $\frac{1}{b}$ such that

$$b \times \frac{1}{b} = 1 = \frac{1}{b} \times b$$

➤ **Distributive property:**

For any three rational numbers a, b and c, $a \times (b + c) = (a \times b) + (a \times c)$

- A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.
- The sum of the first n consecutive odd numbers is n^2 .
- The square of an odd number can be written as the sum of two consecutive natural numbers.
- Mathematically, three numbers a, b and c make a Pythagorean triplet, if the sum of the squares of any two equals the square of the third.
- If n is the number of digits of a number then the numbers of digits of the square root of that

$$\text{number is} = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

- For any two positive numbers a and b, we have

$$(i) \sqrt{ab} = \sqrt{a} \times \sqrt{b} \quad \text{and}$$

$$(ii) \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

- If you multiply a number by itself and then by itself again, the result is a cube number.

➤ **Laws of Exponents:**

$$(i) a^m \times a^n = a^{m+n}$$

$$(ii) \frac{a^m}{a^n} = a^{m-n}$$

$$(iii) (a^m)^n = a^{mn}$$

$$(iv) a^0 = 1$$

$$(v) a^{-m} = \frac{1}{a^m}$$

$$(vi) a^m \times b^m = (ab)^m$$

$$(vii) \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

- To write in scientific notation, we follow the form $N \times 10^a$ where N is a number between 1 and 10, but not 10 itself and a is an integer (positive or negative number).

TRY THESE:

i) Is the number -7 a rational number? Why?

Solution:

-7 is a rational number.

$$-7 = \frac{-7}{1}$$

ii) Write any 6 rational numbers between 0 and 1.

Solution:

$$\frac{3}{5}, \frac{-2}{-5}, \frac{7}{8}, \frac{2}{4}, \frac{1}{3}, \frac{1}{100}$$

WORKED EXAMPLES

1.1. Write the decimal forms of the following rational numbers.

i) $\frac{1}{4}$ ii) $1\frac{3}{20}$ iii) $-5\frac{4}{5}$

iv) 3 v) $\frac{1}{3}$

Solution:

i) $\frac{1}{4} = \frac{25}{100} = 0.25$

ii) $1\frac{3}{20} = \frac{23}{20} = \frac{115}{100} = 1.15$

iii) $-5\frac{4}{5} = \frac{-29}{5} = \frac{-58}{10} = -5.8$

iv) $3 = \frac{3}{1} = \frac{30}{10} = 3.0$

v) $\frac{1}{3} = 0.3333\dots$ (by actual division and it is recurring and non-terminating)

TRY THESE:

I. Write the decimal forms of the following rational numbers.

i) $\frac{4}{5}$ ii) $\frac{6}{25}$ iii) $\frac{486}{1000}$

iv) $\frac{1}{9}$ v) $3\frac{1}{4}$ vi) $-2\frac{3}{5}$

Solution:

i) $\frac{4}{5} = 0.8$

ii) $\frac{6}{25} = 0.24$

iii) $\frac{486}{1000} = 0.486$

iv) $\frac{1}{9} = 0.1111\dots$

v) $3\frac{1}{4} = \frac{13}{4} = 3.25$

vi) $-2\frac{3}{5} = \frac{-13}{5} = -2.6$

II. 1. $\frac{7}{3} = \frac{?}{9} = \frac{49}{?} = \frac{-21}{?}$

2. $\frac{-2}{5} = \frac{?}{10} = \frac{6}{?} = \frac{-8}{?}$

Solution:

i) $\frac{7}{3} = \frac{21}{9} = \frac{49}{21} = \frac{-21}{-9}$

ii) $\frac{-2}{5} = \frac{-4}{10} = \frac{6}{-15} = \frac{-8}{20}$

1.2. Reduce to the standard form

(i) $\frac{48}{-84}$ (ii) $\frac{-18}{-42}$

Solution:

(i) Method 1 :

$$\frac{48}{-84} = \frac{48 \div (-2)}{-84 \div (-2)} = \frac{-24 \div 2}{42 \div 2} = \frac{-12 \div 3}{21 \div 3} = \frac{-4}{7}$$

(dividing by -2 , 2 and 3 successively)

Method 2 :

The HCF of 48 and 84 is 12. Thus, we can get its standard form by dividing it by -12.

$$\frac{48}{-84} = \frac{48 \div (-12)}{-84 \div (-12)} = \frac{-4}{7}$$

(ii) Method 1 :

$$\frac{-18}{-42} = \frac{-18 \div (-2)}{-42 \div (-2)} = \frac{9 \div 3}{21 \div 3} = \frac{3}{7}$$

(dividing by -2 and 3 successively)

Method 2 :

The HCF of 18 and 42 is 6. Thus, we can get its standard form by dividing it by 6.

$$\frac{-18}{-42} = \frac{-18 \times (-1)}{-42 \times (-1)} = \frac{18}{42} = \frac{18 \div 6}{42 \div 6} = \frac{3}{7}$$

TRY THESE:

1. Which of the following pairs represents equivalent rational numbers?

i) $\frac{-6}{4}, \frac{18}{-12}$ ii) $\frac{-4}{-20}, \frac{1}{-5}$

iii) $\frac{-12}{-17}, \frac{60}{85}$

Solution:

i) $\frac{-6}{4} = \frac{18}{-12}$

$$(-6)(-12) = 4 \times 18$$

$$72 = 72$$

This is true.

$\therefore \frac{-6}{4}$ and $\frac{18}{-12}$ are an equivalent rational numbers.

ii) $\frac{-4}{-20} = \frac{1}{-5}$

$$(-4)(-5) = 1 \times (-20)$$

$$20 = -20$$

This is not true.

$\therefore \frac{-4}{-20}$ and $\frac{1}{-5}$ are not an equivalent rational numbers.

iii) $\frac{-12}{-17} = \frac{60}{85}$

$$(-12) \times 85 = 60 \times (-17)$$

$$-1020 = -1020$$

This is true.

$\therefore \frac{-12}{-17}$ and $\frac{60}{85}$ are an equivalent rational numbers.

2. Find the standard form of

i) $\frac{36}{-96}$

ii) $\frac{-56}{-72}$

iii) $\frac{27}{18}$

Solution:

i) $\frac{36}{-96} = \frac{2 \times 2 \times 3 \times 3}{-3 \times 2 \times 2 \times 2 \times 2 \times 2}$

$$= \frac{3}{-2 \times 2 \times 2} = \frac{3}{-8}$$

ii) $\frac{-56}{-72} = \frac{-2 \times 2 \times 2 \times 7}{-2 \times 2 \times 2 \times 3 \times 3} = \frac{7}{9}$

iii) $\frac{27}{18} = \frac{3 \times 3 \times 3}{2 \times 3 \times 3} = \frac{3}{2}$

3. Mark the following rational numbers on a number line.

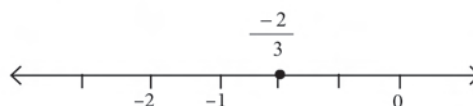
i) $\frac{-2}{3}$

ii) $\frac{-8}{-5}$

iii) $\frac{5}{-4}$

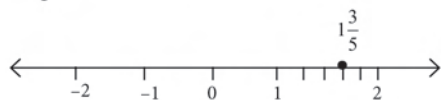
Solution:

i) $\frac{-2}{3}$ lies between 0 and -1



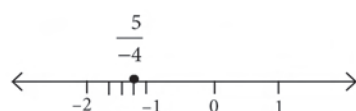
$$\text{ii) } \frac{-8}{-5} = \frac{8}{5} = 1\frac{3}{5}$$

$\therefore \frac{-8}{-5}$ lies between 1 and 2



$$\text{iii) } \frac{5}{-4} = \frac{-5}{4} = -1\frac{1}{4}$$

$\frac{5}{-4}$ lies between -1 and -2.



1.3. Compare $\frac{5}{17}$ and $\frac{-10}{19}$

Solution:

Since every positive number is greater than every negative number, we conclude that

$$\frac{5}{17} > \frac{-10}{19}.$$

1.4 Compare $\frac{1}{3}$ and $\frac{4}{3}$

Solution:

Since the denominators are the same, just compare the numerators.

Since $1 < 4$, we conclude that $\frac{1}{3} < \frac{4}{3}$

1.5 Compare $\frac{3}{4}$ and $\frac{5}{6}$

Solution:

The LCM of the denominators is 12. Consider for each rational number an equivalent rational number with the LCM 12 as denominator.

We get $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$, which become like fractions now.

$$\text{Here, } \frac{9}{12} < \frac{10}{12}.$$

Hence, we conclude that $\frac{3}{4} < \frac{5}{6}$.

1.6 Compare $\frac{9}{-4}$ and $\frac{-2}{3}$

Solution:

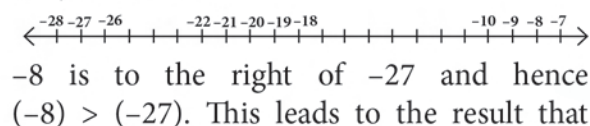
The number $\frac{9}{-4}$ is not in standard form. First put it in the standard form. $\frac{9}{-4} = \frac{9}{-4} \times \frac{-1}{-1}$

(to get a positive denominator = $\frac{-9}{4}$)

Now, we shall compare the fractions $\frac{-9}{4}$ and $\frac{-2}{3}$. We find that these two fractions are unlike fractions. To make them as like fractions, we make use of their LCM, which is 12.

We can now compare their equivalent fractions $\frac{-9}{4} = \frac{-27}{12}$ and $\frac{-2}{3} = \frac{-8}{12}$. We find that the denominators are the same and so just comparing the numerators -27 and -8 are enough.

Visualizing these numbers on the number line, we see that



$\frac{-8}{12} > \frac{-27}{12}$ and consequently we conclude

that $\frac{-2}{3} > \frac{9}{-4}$

1.7 Write the following rational numbers in descending and ascending order.

$$\frac{-3}{5}, \frac{7}{-10}, \frac{-15}{20}, \frac{14}{-30}, \frac{-8}{15}$$

Solution:

First make the denominators to be positive and write the numbers in standard form as $\frac{-3}{5}$, $\frac{-7}{10}$, $\frac{-15}{20}$, $\frac{-14}{30}$, $\frac{-8}{15}$. Here the LCM of 5, 10, 15, 20 and 30 is 60. Change the given rational numbers equivalent form with common denominator 60.

$$\frac{-3}{5} = \frac{-3}{5} \times \frac{12}{12} = \frac{-36}{60}$$

$$\frac{7}{-10} = \frac{-7}{10} \times \frac{6}{6} = \frac{-42}{60}$$

$$\frac{-15}{20} = \frac{-15}{20} \times \frac{3}{3} = \frac{-45}{60}$$

$$\frac{-14}{30} = \frac{-14}{30} \times \frac{2}{2} = \frac{-28}{60}$$

$$\frac{-8}{15} = \frac{-8}{15} \times \frac{4}{4} = \frac{-32}{60}$$

Now, comparing the numerators alone, that is $-36, -42, -45, -28$ and -32 we see that $-45 < -42 < -36 < -32 < -28$

$$\text{Hence, } \frac{-45}{60} < \frac{-42}{60} < \frac{-36}{60} < \frac{-32}{60} < \frac{-28}{60}$$

$$\text{and so, } \frac{-15}{20} < \frac{7}{-10} < \frac{-3}{5} < \frac{-8}{15} < \frac{-14}{30}$$

So, the ascending order is

$$\frac{-15}{20}, \frac{7}{-10}, \frac{-3}{5}, \frac{-8}{15} \text{ and } \frac{-14}{30}.$$

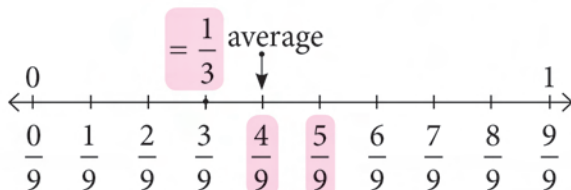
Also, its reverse order gives the descending order as $\frac{-14}{30}, \frac{-8}{15}, \frac{-3}{5}, \frac{7}{-10}$ and $\frac{-15}{20}$.

1.8 Find a rational number between $\frac{1}{3}$ and $\frac{5}{9}$.

Solution:

The average of $\frac{1}{3}$ and $\frac{5}{9}$

$$\begin{aligned} &= \frac{1}{2} \left(\frac{1}{3} + \frac{5}{9} \right) = \frac{1}{2} \left(\frac{1}{3} \times \frac{3}{3} + \frac{5}{9} \right) \\ &= \frac{1}{2} \left(\frac{3}{9} + \frac{5}{9} \right) = \frac{1}{2} \times \frac{8}{9} = \frac{4}{9} \end{aligned}$$



THINK:

Are there any rational numbers between

$$\frac{-7}{11} \text{ and } \frac{6}{-11}?$$

Ans: Yes. The average of $\frac{-7}{11}$ and $\frac{-6}{11}$

$$= \frac{1}{2} \left(\frac{-7}{11} + \frac{-6}{11} \right) = \frac{1}{2} \left(\frac{-13}{11} \right) = \frac{-13}{22}$$

The rational numbers between $\frac{-7}{11}$ and $\frac{-6}{11}$ is $\frac{-13}{22}$.

1.9 Find atleast two rational numbers between

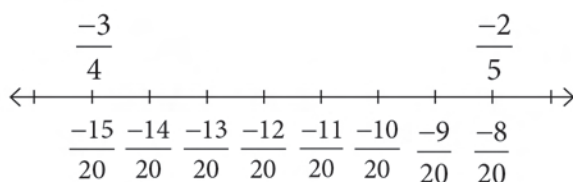
$$\frac{-3}{4} \text{ and } \frac{-2}{5}$$

Solution:

The denominators are different for the given rational numbers. The LCM of the denominators 4 and 5 is 20. Make the rational numbers such that they have common denominators as 20. Here,,

$$\frac{-3}{4} = \frac{-3}{4} \times \frac{5}{5} = \frac{-15}{20} \text{ and } \frac{-2}{5} = \frac{-2}{5} \times \frac{4}{4} = \frac{-8}{20}$$

It is easy now to find and insert rational numbers between $\frac{-15}{20}$ and $\frac{-8}{20}$ as shown below.



We can list a few rational numbers as

$\frac{-14}{20}$, $\frac{-13}{20}$, $\frac{-12}{20}$, $\frac{-11}{20}$, $\frac{-10}{20}$ and $\frac{-9}{20}$ between

$\frac{-15}{20}$ and $\frac{-8}{20}$.

EXERCISE 1.1

1. Fill in the blanks:

- i) $\frac{-19}{5}$ lies between the integers _____ and _____. **Ans: -4 and -3**

Solution: $\frac{-19}{5} = -3.8$

- ii) The decimal form of the rational number $\frac{15}{-4}$ is _____. **Ans: -3.75**

- iii) The rational numbers $\frac{-8}{3}$ and $\frac{8}{3}$ are equidistant from _____. **Ans: 0**

- iv) The next rational number in the sequence $\frac{-15}{24}$, $\frac{20}{-32}$, $\frac{-25}{40}$ is _____. **Ans: $\frac{30}{-48}$**

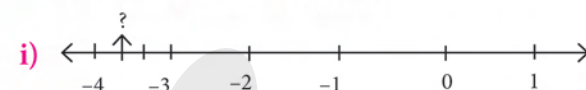
- v) The standard form of $\frac{+58}{-78}$ is _____. **Ans: $\frac{-29}{39}$**

2. Say True or False:

- i) 0 is the smallest rational number. **Ans: False**
Reason:
 Negative integers are also rational numbers.
- ii) $\frac{-4}{5}$ lies to the left of $\frac{-3}{4}$ **Ans: True**
- iii) $\frac{-19}{5}$ is greater than $\frac{15}{-4}$ **Ans: False**
- iv) The average of two rational numbers lies between them. **Ans: True**

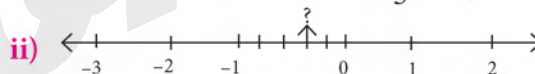
- v) There are an unlimited number of rational numbers between 10 and 11. **Ans: True**

3. Find the rational numbers represented by each of the question mark marked on the following number lines.



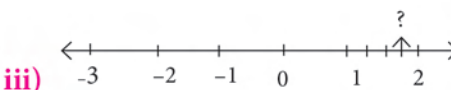
Solution:

The rational number for the point marked on the number line is $-3\frac{2}{3} = \frac{-11}{3}$



Solution:

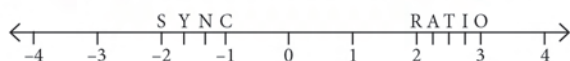
The rational number for the point marked on the number line is $\frac{-2}{5}$



Solution:

The rational number for the point marked on the number line is $1\frac{3}{4} = \frac{7}{4}$

4. The points S, Y, N, C, R, A, T, I and O on the number line are such that CN=NY=YS and RA=AT=TI=IO. Find the rational numbers represented by the letters Y, N, A, T and I.



Solution:

Take SYNC:

CS is divided into 3 parts.

$$\therefore \text{C point} \rightarrow -1 \times \frac{3}{3} = \frac{-3}{3}$$

$$\text{S point} \rightarrow -2 \times \frac{3}{3} = \frac{-6}{3}$$

From these, we get $Y = \frac{-5}{3}$, $N = \frac{-4}{3}$

Now take RATIO

RO is divided into 4 parts.

$$\text{R point} \rightarrow 2 \times \frac{4}{4} = \frac{8}{4}$$

$$\text{O point} \rightarrow 3 \times \frac{4}{4} = \frac{12}{4}$$

From these, we get $A = \frac{9}{4}$, $T = \frac{10}{4}$, $I = \frac{11}{4}$

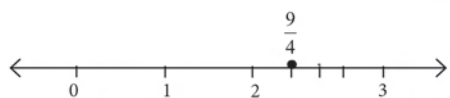
5. Draw a number line and represent the following rational numbers on it.

i) $\frac{9}{4}$ ii) $\frac{-8}{3}$ iii) $\frac{-17}{-5}$ iv) $\frac{15}{-4}$

Solution:

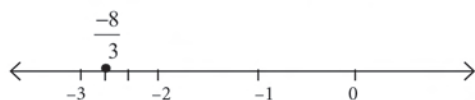
$$\text{i) } \frac{9}{4} = 2\frac{1}{4}$$

$\frac{9}{4}$ lies between 2 and 3



$$\text{ii) } \frac{-8}{3} = -2\frac{2}{3}$$

$\therefore \frac{-8}{3}$ lies between -2 and -3.



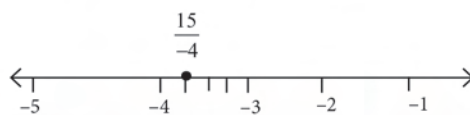
$$\text{iii) } \frac{-17}{-5} = \frac{17}{5} = 3\frac{2}{5}$$

$\therefore \frac{-17}{-5}$ lies between 3 and 4.



$$\text{iv) } \frac{15}{-4} = \frac{-15}{4} = -3\frac{3}{4}$$

$\frac{15}{-4}$ lies between -3 and -4.



6. Write the decimal form of the following rational numbers.

i) $\frac{1}{11}$

ii) $\frac{-2}{5}$

iii) $\frac{-18}{7}$

iv) $1\frac{2}{5}$

v) $-3\frac{1}{2}$

Solution:

$$\text{i) } \frac{1}{11} = 0.09090.....$$

$$\text{ii) } \frac{-2}{5} = 3.25$$

$$\text{iii) } \frac{-18}{7} = -2.571$$

$$\text{iv) } 1\frac{2}{5} = \frac{7}{5} = 1.4$$

$$\text{v) } -3\frac{1}{2} = \frac{-7}{2} = 3.5$$

7. List any five rational numbers between the given rational numbers

i) -2 and 0

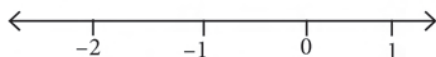
ii) $\frac{-1}{2}$ and $\frac{3}{5}$

iii) $\frac{1}{4}$ and $\frac{7}{20}$

iv) $\frac{-6}{4}$ and $\frac{-23}{10}$

Solution:

- i) -2 and 0



The five rational numbers between -2 and 0 are -0.5, -0.10, -1, -1.5, -1.8

That is $\frac{-5}{10}, \frac{-1}{10}, -1, \frac{-15}{10}, \frac{-18}{10}$

- ii) $\frac{-1}{2}$ and $\frac{3}{5}$

The five rational numbers between $\frac{-1}{2}$ and $\frac{3}{5}$ are 0, $\frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}$

- iii) $\frac{1}{4}$ and $\frac{7}{20}$

The five rational numbers between $\frac{1}{4} = 0.25$ and $\frac{7}{20} = 0.35$ are 0.26, 0.27, 0.28, 0.29 and 0.3 that is $\frac{26}{100}, \frac{27}{100}, \frac{28}{100}, \frac{29}{100}, \frac{30}{100}$

- iv) $\frac{-6}{4}$ and $\frac{-23}{10}$

The five rational numbers between $\frac{-6}{4} = -1.5$ and $\frac{-23}{10} = -2.3$ are -1.6, -1.7, -1.8, -1.9, -2.0, that is $\frac{-16}{10}, \frac{-17}{10}, \frac{-18}{10}, \frac{-19}{10}, \frac{-20}{10}$

8. Use the method of averages to write 2 rational numbers between $\frac{14}{5}$ and $\frac{16}{3}$.

Solution:

Let a and b be $\frac{14}{5}$ and $\frac{16}{3}$

We know that the average of a and b is

$$\frac{1}{2} (a + b)$$

The average of a and b is C_1

$$\begin{aligned} C_1 &= \frac{1}{2} (a+b) = \frac{1}{2} \left(\frac{14}{5} + \frac{16}{3} \right) \\ &= \frac{1}{2} \left(\frac{42+80}{15} \right) = \frac{1}{2} \left(\frac{122}{15} \right) = \frac{61}{15} \\ \therefore \frac{14}{5} &< \frac{61}{15} < \frac{16}{3} \dots\dots\dots (1) \end{aligned}$$

The average of $\frac{14}{5}$ and $\frac{61}{15}$ is C_2

$$\begin{aligned} C_2 &= \frac{1}{2} \left(\frac{14}{5} + \frac{61}{15} \right) = \frac{1}{2} \left(\frac{42+61}{15} \right) \\ &= \frac{1}{2} \left(\frac{103}{15} \right) = \frac{103}{30} \\ \therefore \frac{14}{5} &< \frac{103}{30} < \frac{61}{15} \dots\dots\dots (2) \end{aligned}$$

The average of $\frac{14}{5}$ and $\frac{103}{30}$ is C_3

$$\begin{aligned} C_3 &= \frac{1}{2} \left(\frac{14}{5} + \frac{103}{30} \right) = \frac{1}{2} \left(\frac{84+103}{30} \right) \\ &= \frac{1}{2} \left(\frac{187}{30} \right) = \frac{187}{60} \\ \therefore \frac{14}{5} &< \frac{187}{60} < \frac{103}{30} \dots\dots\dots (3) \end{aligned}$$

From 1, 2 and 3 we get

$$\frac{14}{5} < \frac{187}{60} < \frac{103}{30} < \frac{61}{15} < \frac{16}{3}$$

9. Compare the following pairs of rational numbers.

- i) $\frac{-11}{5}, \frac{-21}{8}$ ii) $\frac{3}{-4}, \frac{-1}{2}$ iii) $\frac{2}{3}, \frac{4}{5}$

Solution:

i) $\frac{-11}{5}, \frac{-21}{8}$

The LCM of 5 and 8 is 40

$$\frac{-11}{5} = \frac{-88}{40} \text{ and } \frac{-21}{8} = \frac{-105}{40}$$

$$\frac{-105}{40} < \frac{-88}{40} \quad (\text{ie}) \quad \frac{-21}{8} < \frac{-11}{5}$$

ii) $\frac{3}{-4}, \frac{-1}{2}$

The LCM of 2 and 4 is 4.

$$\frac{3}{-4} = \frac{-3}{4} \text{ and } \frac{-1}{2} = \frac{-2}{4}$$

$$\frac{-3}{4} < \frac{-2}{4} \quad (\text{ie}) \quad \frac{-3}{4} < \frac{-1}{2}$$

iii) $\frac{2}{3}, \frac{4}{5}$

The LCM of 3 and 5 is 15.

$$\frac{2}{3} = \frac{10}{15} \text{ and } \frac{4}{5} = \frac{12}{15}$$

$$\therefore \frac{12}{15} > \frac{10}{15} \quad (\text{ie}) \quad \frac{4}{5} > \frac{2}{3}$$

10. Arrange the following rational numbers in ascending and descending order.

i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$

ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

Solution:

i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$

The LCM of 12, 8, 24, 9 and 36 is 72

$$\frac{-5}{12} = \frac{-30}{72}$$

$$\frac{-11}{8} = \frac{-99}{72}$$

$$\frac{-15}{24} = \frac{-45}{72}$$

$$\frac{-7}{-9} = \frac{7}{9} = \frac{56}{72}$$

$$\frac{12}{36} = \frac{24}{72}$$

$$\frac{-99}{72} < \frac{-45}{72} < \frac{-30}{72} < \frac{24}{72} < \frac{56}{72}$$

The ascending order of the given numbers is

$$\frac{-11}{8} < \frac{-15}{24} < \frac{-5}{12} < \frac{12}{36} < \frac{-7}{-9}$$

The descending order of the given numbers is

$$\frac{-7}{-9} > \frac{12}{36} > \frac{-5}{12} > \frac{-15}{24} > \frac{-11}{8}$$

ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

The LCM of 10, 5, 4 and 20 is 20

$$\frac{-17}{10} = \frac{-34}{20}$$

$$\frac{-7}{5} = \frac{-28}{20}$$

$$\frac{-2}{4} = \frac{-10}{20}$$

$$\frac{-19}{20} = \frac{-19}{20}$$

$$\frac{-34}{20} < \frac{-28}{20} < \frac{-19}{20} < \frac{-10}{20} < 0$$

$$\frac{-17}{10} < \frac{-7}{5} < \frac{-19}{20} < \frac{-2}{4} < 0$$

 $\therefore$ The ascending order of the given numbers is

$$\frac{-17}{10} < \frac{-7}{5} < \frac{-19}{20} < \frac{-2}{4} < 0$$

 $\therefore$ The descending order of the given numbers is

$$0 > \frac{-2}{4} > \frac{-19}{20} > \frac{-7}{5} > \frac{-17}{10}$$

OBJECTIVE TYPE QUESTIONS

11. The number which is subtracted from $\frac{-6}{11}$ to get $\frac{8}{9}$ is

a) $\frac{34}{99}$ b) $\frac{-142}{99}$ c) $\frac{142}{99}$ d) $\frac{-34}{99}$

Ans: b) $\frac{-142}{99}$

Solution:

$$\frac{-6}{11} - x = \frac{8}{9}$$

$$x = \frac{-6}{11} - \frac{8}{9} = \frac{-54 - 88}{99} = \frac{-142}{99}$$

12. Which of the following pairs is equivalent?

a) $\frac{-20}{12}, \frac{5}{3}$ b) $\frac{16}{-30}, \frac{-8}{15}$
c) $\frac{-18}{36}, \frac{-20}{40}$ d) $\frac{7}{-5}, \frac{-5}{7}$

Ans: b) $\frac{16}{-30}, \frac{-8}{15}$

Solution:

$$\frac{16}{-30} = \frac{-8}{15}$$

$$240 = 240$$

13. $\frac{-5}{4}$ is a rational number which lies between

a) 0 and $\frac{-5}{4}$ b) -1 and 0
c) -1 and -2 d) -4 and -5

Ans: (c) -1 and -2

Solution:

$$\frac{-5}{4} = -1\frac{1}{4}$$

$-1\frac{1}{4}$ lies between -1 and -2.

14. Which of the following rational numbers is the greatest?

a) $\frac{-17}{24}$ b) $\frac{-13}{16}$ c) $\frac{7}{-8}$ d) $\frac{-31}{32}$

Ans: a) $\frac{-17}{24}$

Solution:

The LCM of 24, 16, 8 and 32 is 96

$$\frac{-17}{24} = \frac{-68}{96}, \frac{-13}{16} = \frac{-78}{96},$$

$$\frac{7}{-8} = \frac{-84}{96}, \frac{-31}{32} = \frac{-93}{96}$$

The greatest number is $\frac{-68}{96}$

ie) $\frac{-17}{24}$

15. The sum of the digits of the denominator in the simplest form of $\frac{112}{528}$ is _____.

a) 4 b) 5 c) 6 d) 7

Ans: c) 6

Solution:

$$\begin{aligned} \text{Sum of the digits of the denominator} \\ &= 5 + 2 + 8 \\ &= 15 = 1 + 5 \\ &= 6 \end{aligned}$$

WORKED EXAMPLES

1.10 Add: $\frac{-6}{11}, \frac{8}{11}, \frac{-12}{11}$

Solution:

Write the given rational numbers in the standard form and then add them.

$$\text{So, } \frac{-6}{11} + \frac{8}{11} + \frac{-12}{11} = \frac{-6+8-12}{11} = \frac{-10}{11}$$

1.11 Add: $\frac{-5}{9}, \frac{-4}{3}, \frac{7}{12}$

Solution:

LCM of 9, 3, 12 = 36

$$\begin{aligned} & \frac{-5}{9} + \frac{-4}{3} + \frac{7}{12} \\ &= \frac{-5}{9} \times \frac{4}{4} + \frac{-4}{3} \times \frac{12}{12} + \frac{7}{12} \times \frac{3}{3} \\ &= \frac{-20}{36} + \frac{-48}{36} + \frac{21}{36} \\ &= \frac{-20 - 48 + 21}{36} = \frac{-47}{36} \end{aligned}$$

1.12 Subtract: $\frac{9}{17}$ from $\frac{-12}{17}$

Solution:

$$\text{Now, } \frac{-12}{17} - \frac{9}{17} = \frac{-12-9}{17} = \frac{-21}{17}.$$

1.13 Subtract: $\left(-2\frac{6}{11}\right)$ from $\left(-4\frac{5}{22}\right)$

Solution:

$$\begin{aligned} \text{Now, } & \left(-4\frac{5}{22}\right) - \left(-2\frac{6}{11}\right) \\ &= \frac{-93}{22} - \left(\frac{-28}{11}\right) \\ &= \frac{-93}{22} + \frac{28}{11} \\ &= \frac{-93 + 28 \times 2}{22} \\ &= \frac{-93 + 56}{22} = \frac{-37}{22} = -1\frac{15}{22}. \end{aligned}$$

1.14 Find : (i) $\frac{-5}{8} \times \frac{7}{3}$ (ii) $\frac{-6}{-11} \times (-4)$

Solution:

$$(i) \frac{-5}{8} \times \frac{7}{3} = \frac{-5 \times 7}{8 \times 3} = \frac{-35}{24}$$

$$\begin{aligned} (ii) \frac{-6}{-11} \times (-4) &= \frac{6}{11} \times \frac{(-4)}{1} \\ &= \frac{6 \times (-4)}{11 \times 1} = \frac{-24}{11} \end{aligned}$$

1.15 Divide : $\frac{7}{-8}$ by $\frac{-3}{4}$

Solution:

$$\frac{7}{-8} \div \frac{-3}{4} = \frac{-7}{8} \times \frac{-4}{3} = \frac{7}{6}.$$

TRY THESE:

Divide:

i) $\frac{-7}{3}$ by 5 ii) 5 by $\frac{-7}{3}$ iii) $\frac{-7}{3}$ by $\frac{35}{6}$

Solution:

i) $\frac{-7}{3}$ by 5

$$\frac{-7}{3} \div 5 = \frac{-7}{3} \times \frac{1}{5} = \frac{-7}{15}.$$

ii) 5 by $\frac{-7}{3}$

$$5 \div \frac{-7}{3} = 5 \times \frac{-3}{7} = \frac{-15}{7}$$

iii) $\frac{-7}{3}$ by $\frac{35}{6}$

$$5 \div \frac{-7}{3} = \frac{35}{6} = \frac{\cancel{35}^7}{\cancel{6}_2} \times \frac{\cancel{6}_2^1}{\cancel{35}_5} = \frac{-2}{5}$$

1.16 The sum of two rational numbers is $\frac{4}{5}$. If one number is $\frac{2}{15}$, then find the other.

Solution:

Let the other number be x .

$$\text{Given, } \frac{2}{15} + x = \frac{4}{5}$$

$$\Rightarrow x = \frac{4}{5} - \frac{2}{15} = \frac{12-2}{15} = \frac{10}{15}$$

$$x = \frac{2}{3}$$

1.17 The product of two rational numbers is $-\frac{2}{3}$.

If one number is $\frac{3}{7}$, then find the other.

Solution:

Let the other number be x .

$$\text{Given, } \frac{3}{7}x = -\frac{2}{3}$$

Multiplying by the reciprocal of $\frac{3}{7}$,
that is $\frac{7}{3}$ on both sides,

$$\therefore \frac{7}{3} \times \frac{3}{7} \times x = \frac{7}{3} \times -\frac{2}{3}$$

$$x = \frac{-14}{9}$$

1.18 One roll of ribbon is $18\frac{3}{4}$ m long. Sankari

has four full rolls and one – third of another roll. How many metres of ribbon does Sankari have in total?

Solution:

Number of metres of ribbon Sankari has in total

$$= 18\frac{3}{4} \times 4\frac{1}{3}$$

$$= \frac{75}{4} \times \frac{13}{3} = \frac{325}{4}$$

$$= 81\frac{1}{4} \text{ m.}$$

1.19 Find the rational numbers that should be added and subtracted so that they will make the sum $3\frac{1}{2} + 1\frac{3}{4} + 2\frac{3}{8}$ to the nearest whole number.

Solution:

$$\text{Now, } 3\frac{1}{2} + 1\frac{3}{4} + 2\frac{3}{8}$$

$$= \frac{7}{2} + \frac{7}{4} + \frac{19}{8} = \frac{7 \times 4 + 7 \times 2 + 19 \times 1}{8}$$

$$= \frac{28 + 14 + 19}{8} = \frac{61}{8}$$

$$= 7\frac{5}{8}$$

which lies between the whole numbers 7 and 8.

If we subtract $\frac{5}{8}$ from $7\frac{5}{8}$, it becomes 7.

If we add $\frac{3}{8}$ to $7\frac{5}{8}$, it becomes

$$7 + \frac{5}{8} + \frac{3}{8} = 7 + 1 = 8.$$

1.20 A student instead of multiplying a number by $\frac{8}{9}$, by mistake divided it by $\frac{8}{9}$. If the difference between the correct answer and the answers got by him is 34, then find the number.

Solution:

Let the number be x .

The student had to find $\frac{8x}{9}$ but,

he had found $\left(\frac{x}{\frac{8}{9}}\right)$, that is $\frac{9x}{8}$

$$\text{Now, } \frac{9x}{8} - \frac{8x}{9} = 34 \text{ (given)}$$

$$\frac{81x - 64x}{72} = 34 \Rightarrow \frac{17x}{72} = 34$$

$$x = \frac{34 \times 72}{17} = 144$$

1.21 Simplify:

$$\left(\frac{4}{3} - \left(-\frac{3}{2}\right)\right) + \left(-\frac{5}{3} \div \frac{30}{12}\right) + \left(\frac{-12}{9} \times \frac{-27}{16}\right)$$

Solution:

$$\begin{aligned}
 & \left(\frac{4}{3} - \left(\frac{-3}{2} \right) \right) + \left(\frac{-5}{3} \div \frac{30}{12} \right) + \left(\frac{-12}{9} \times \frac{-27}{16} \right) \\
 &= \left(\frac{4}{3} + \frac{3}{2} \right) + \left(\frac{-5}{3} \times \frac{12}{30} \right) + \left(\frac{-12}{9} \times \frac{-27}{16} \right) \\
 &= \left(\frac{8}{6} + \frac{9}{6} \right) + \left(\frac{-1}{1} \times \frac{4}{6} \right) + \left(\frac{-3}{1} \times \frac{-3}{4} \right)
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\frac{17}{6} \right) + \left(\frac{-4}{6} \right) + \left(\frac{9}{4} \right) \\
 &= \left(\frac{17-4}{6} \right) + \frac{9}{4} \\
 &= \frac{13}{6} + \frac{9}{4} \\
 &= \frac{26+27}{12} = \frac{53}{12}
 \end{aligned}$$

EXERCISE 1.2

1. Fill in the blanks:

i) The value of $\frac{-5}{12} + \frac{7}{15} =$ _____.

Ans: $\frac{1}{20}$

Solution:

The LCM of 12 and 15 is 60

$$\begin{aligned}
 \frac{-5}{12} + \frac{7}{15} &= \frac{-25}{60} + \frac{28}{60} \\
 &= \frac{-25+28}{60} = \frac{3}{60} = \frac{1}{20}
 \end{aligned}$$

ii) The value of $\left(\frac{-3}{6} \right) \times \left(\frac{18}{-9} \right)$ is _____.

Ans: 1

Solution:

$$\frac{-3}{6} \times \frac{18}{-9} = \frac{-54}{-54} = 1$$

iii) The value of $\left(\frac{-15}{23} \right) \div \left(\frac{30}{-46} \right)$ is _____.

Ans: 1

Solution:

$$\begin{aligned}
 & \left(\frac{-15}{23} \right) \div \left(\frac{+30}{-46} \right) \\
 &= \left(\frac{-15}{23} \right) \times \left(\frac{-46}{30} \right) = \left(\frac{2}{2} \right) = 1
 \end{aligned}$$

iv) The rational number _____ does not have a reciprocal. **Ans:** 0

v) The multiplicative inverse of -1 is _____. **Ans:** -1

2. Say True or False:

i) All rational numbers have an additive inverse. **Ans:** True

ii) The rational numbers that are equal to their additive inverses are 0 and -1. **Ans:** False

iii) The additive inverse of $\frac{-11}{-17}$ is $\frac{11}{17}$. **Ans:** False

iv) The rational number which is its own reciprocal is -1. **Ans:** True

v) The multiplicative inverse exists for all rational numbers. **Ans:** False

3. Find the sum :

i) $\frac{7}{5} + \frac{3}{5}$

ii) $\frac{7}{5} + \frac{5}{7}$

iii) $\frac{6}{5} + \left(\frac{-14}{15} \right)$

iv) $-4\frac{2}{3} + 7\frac{5}{12}$

Solution:

i) $\frac{7}{5} + \frac{3}{5}$

Write the given rational numbers in the standard form and then add them.

$$\text{So, } \frac{7}{5} + \frac{3}{5} = \frac{7+3}{5} = \frac{10}{5} = 2$$

ii) $\frac{7}{5} + \frac{5}{7}$

LCM of 5, 7 = 35

$$\begin{aligned} \text{So, } \frac{7}{5} + \frac{5}{7} &= \frac{7 \times 7}{5 \times 7} + \frac{5 \times 5}{7 \times 5} \\ &= \frac{49}{35} + \frac{25}{35} = \frac{49+25}{35} = \frac{74}{35} \end{aligned}$$

iii) $\frac{6}{5} + \left(\frac{-14}{15}\right)$

LCM of 5, 15 = 15

$$\begin{aligned} \text{So, } \frac{6}{5} + \left(\frac{-14}{15}\right) &= \frac{6 \times 3}{5 \times 3} + \frac{-14 \times 1}{15 \times 1} \\ &= \frac{18}{15} + \frac{-14}{15} = \frac{18-14}{15} = \frac{4}{15} \end{aligned}$$

iv) $-4\frac{2}{3} + 7\frac{5}{12}$

$$-4\frac{2}{3} + 7\frac{5}{12} = \frac{-14}{3} + \frac{89}{12}$$

LCM of 3, 12 = 12

$$\begin{aligned} \text{So, } \frac{-14 \times 4}{3 \times 4} + \frac{89 \times 1}{12 \times 1} &= \frac{-56}{12} + \frac{89}{12} \\ &= \frac{-56+89}{12} \\ &= \frac{33}{12} = \frac{11}{4} \\ &= 2\frac{3}{4} \end{aligned}$$

4. Subtract $\frac{-8}{44}$ from $\frac{-17}{11}$

Solution:

$$\frac{-17}{11} - \left(\frac{-8}{44}\right) = \frac{-17}{11} + \frac{8}{44}$$

The LCM of 11 and 44 is 44

$$\begin{aligned} &= \frac{-17 \times 4}{11 \times 4} + \frac{8 \times 1}{44 \times 1} \\ &= \frac{-68}{44} + \frac{8}{44} \\ &= \frac{-68+8}{44} = \frac{-60}{44} \\ &= \frac{-15}{11} \end{aligned}$$

5. Evaluate :

i) $\frac{9}{132} \times \frac{-11}{3}$

ii) $\frac{-7}{27} \times \frac{24}{-35}$

Solution:

i) $\frac{9}{132} \times \frac{-11}{3} = \frac{3}{12} \times \frac{-1}{1} = \frac{-1}{4}$

ii) $\frac{-7}{27} \times \frac{24}{-35} = \frac{-1}{9} \times \frac{8}{-5} = \frac{8}{45}$

6. Divide:

i) $\frac{-21}{5}$ by $\frac{-7}{-10}$

ii) $\frac{-3}{13}$ by -3

iii) -2 by $\frac{-6}{15}$

Solution:

i) $\frac{-21}{5} \div \frac{-7}{-10} = \frac{-21}{5} \times \frac{10}{7}$
 $= \frac{-3}{1} \times \frac{2}{1} = -6$

ii) $\frac{-3}{13} \div -3 = \frac{-3}{13} \times \frac{1}{-3} = \frac{1}{13}$

iii) $-2 \div \frac{-6}{15} = -2 \times \frac{15}{-6}$
 $= 1 \times \frac{15}{3} = 5$

7. Find $(a + b) \div (a - b)$ if

i) $a = \frac{1}{2}, b = \frac{2}{3}$ ii) $a = \frac{-3}{5}, b = \frac{2}{15}$

Solution:

i) $a = \frac{1}{2}, b = \frac{2}{3}$

$$(a + b) \div (a - b) = \left(\frac{1}{2} + \frac{2}{3}\right) \div \left(\frac{1}{2} - \frac{2}{3}\right)$$

LCM of 2, 3 = 6

$$= \left(\frac{1 \times 3}{2 \times 3} + \frac{2 \times 2}{3 \times 2}\right) \div \left(\frac{1 \times 3}{2 \times 3} - \frac{2 \times 2}{3 \times 2}\right)$$

$$= \left(\frac{3}{6} + \frac{4}{6}\right) \div \left(\frac{3}{6} - \frac{4}{6}\right)$$

$$= \left(\frac{3+4}{6}\right) \div \left(\frac{3-4}{6}\right)$$

$$= \frac{7}{6} \div \frac{-1}{6} = \frac{7}{6} \times \frac{6}{-1} = \frac{7}{-1} = -7$$

ii) $a = \frac{-3}{5}, b = \frac{2}{15}$

$$(a + b) \div (a - b) = \left(\frac{-3}{5} + \frac{2}{15}\right) \div \left(\frac{-3}{5} - \frac{2}{15}\right)$$

LCM of 5, 15 = 15

$$= \left(\frac{-3 \times 3}{5 \times 3} + \frac{2 \times 1}{15 \times 1}\right) \div \left(\frac{-3 \times 3}{5 \times 3} - \frac{2 \times 1}{15 \times 1}\right)$$

$$= \left(\frac{-9}{15} + \frac{2}{15}\right) \div \left(\frac{-9}{15} - \frac{2}{15}\right)$$

$$= \left(\frac{-9+2}{15}\right) \div \left(\frac{-9-2}{15}\right)$$

$$= \frac{-7}{15} \div \frac{-11}{15} = \frac{-7}{15} \times \frac{15}{-11} = \frac{7}{11}$$

8. Simplify : $\frac{1}{2} + \left(\frac{3}{2} - \frac{2}{5}\right) \div \frac{3}{10} \times 3$ and show that it is a rational number between 11 and 12.

Solution:

$$\frac{1}{2} + \left(\frac{3}{2} - \frac{2}{5}\right) \div \frac{3}{10} \times 3$$

$$= \frac{1}{2} + \left(\frac{3 \times 5}{2 \times 5} - \frac{2 \times 2}{5 \times 2}\right) \div \frac{3}{10} \times 3$$

$$= \frac{1}{2} + \left(\frac{15}{10} - \frac{4}{10}\right) \times \frac{10}{3} \times 3$$

$$= \frac{1}{2} + \frac{11}{10} \times 10 = \frac{1}{2} + 11$$

$$= \frac{1}{2} + \frac{11 \times 2}{1 \times 2} = \frac{1}{2} + \frac{22}{2}$$

$$= \frac{23}{2} = 11 \frac{1}{2} \text{ which lies between 11 and}$$

12. Hence verified.

9. Simplify :

i) $\left[\frac{11}{8} \times \left(\frac{-6}{33}\right)\right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20}\right)\right] - \left[\frac{4}{7} \times \frac{-7}{5}\right]$

ii) $\left[\frac{4}{3} \div \left(\frac{8}{-7}\right)\right] - \left[\frac{3}{4} \times \frac{4}{3}\right] + \left[\frac{4}{3} \times \left(\frac{-1}{4}\right)\right]$

Solution:

i) $\left[\frac{11}{8} \times \left(\frac{-6}{33}\right)\right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20}\right)\right] - \left[\frac{4}{7} \times \frac{-7}{5}\right]$

$$= \left[\frac{11}{8} \times \frac{-6}{33}\right] + \left[\frac{1}{3} + \left(\frac{3}{5} \times \frac{20}{9}\right)\right] - \left[\frac{-4}{5}\right]$$

$$= \left[\frac{1}{8} \times \frac{-6}{3}\right] + \left[\frac{1}{3} + \frac{4}{3}\right] - \left[\frac{-4}{5}\right]$$

$$= \frac{-1}{4} + \frac{5}{3} + \frac{4}{5}$$

LCM of 4, 3, 5 = 60

$$= \frac{-1}{4} + \frac{5}{3} + \frac{4}{5}$$

$$= \frac{-1 \times 15}{4 \times 15} + \frac{5 \times 20}{3 \times 20} + \frac{4 \times 12}{5 \times 12}$$

$$= \frac{-15}{60} + \frac{100}{60} + \frac{48}{60}$$

$$= \frac{133}{60}$$

$$\begin{aligned} \text{ii) } & \left[\frac{4}{3} \div \left(\frac{8}{-7} \right) \right] - \left[\frac{3}{4} \times \frac{4}{3} \right] + \left[\frac{4}{3} \times \left(\frac{-1}{4} \right) \right] \\ &= \left[\frac{4}{3} \times \left(\frac{-7}{8} \right) \right] - \left[\frac{12}{12} \right] + \frac{-1}{3} \\ &= \frac{-7}{6} - 1 + \frac{-1}{3} \end{aligned}$$

LCM of 3, 6 = 6

$$\begin{aligned} &= \frac{-7 \times 1}{6 \times 1} - \frac{1 \times 6}{1 \times 6} + \frac{-1 \times 2}{3 \times 2} \\ &= \frac{-7}{6} - \frac{6}{6} - \frac{2}{6} \\ &= \frac{-7-6-2}{6} = \frac{-15}{6} = \frac{-5}{2} \end{aligned}$$

10. A student had multiplied a number by $\frac{4}{3}$ instead of dividing it by $\frac{4}{3}$ and got 70 more than the correct answer. Find the number. **Ans: 120**

Solution:

Let the number be x .

The student had to find $\frac{4x}{3}$ but he had

found $\frac{x}{4/3}$ that is $\frac{3x}{4}$

$$\text{Now, } \frac{4x}{3} - \frac{3x}{4} = 70 \text{ (given)}$$

$$\frac{-9x + 16x}{12} = 70$$

$$\frac{7x}{12} = 70$$

$$\begin{aligned} x &= \frac{70 \times 12}{7} \\ &= 120 \end{aligned}$$

OBJECTIVE TYPE QUESTIONS

11. The standard form of the sum

$$\frac{3}{4} + \frac{5}{6} + \left(\frac{-7}{12} \right) \text{ is}$$

- a) 1 b) $\frac{-1}{2}$ c) $\frac{1}{12}$ d) $\frac{1}{22}$

Ans: a) 1

Solution:

$$\frac{3}{4} + \frac{5}{6} + \left(\frac{-7}{12} \right) = \frac{9+10-7}{12} = 1$$

12. $\left(\frac{3}{4} - \frac{5}{8} \right) + \frac{1}{2} =$ _____.

- a) $\frac{15}{64}$ b) 1 c) $\frac{5}{8}$ d) $\frac{1}{16}$

Solution:

Ans: c) $\frac{5}{8}$

$$\frac{6-5}{8} + \frac{1}{2} = \frac{1}{8} + \frac{1}{2} = \frac{1+4}{8} = \frac{5}{8}$$

13. $\frac{3}{4} \div \left(\frac{5}{8} + \frac{1}{2} \right) =$ _____.

- a) $\frac{13}{10}$ b) $\frac{2}{3}$ c) $\frac{3}{2}$ d) $\frac{5}{8}$

Ans: b) $\frac{2}{3}$

Solution:

$$\begin{aligned} \frac{3}{4} \div \left(\frac{5}{8} + \frac{1}{2} \right) &= \frac{3}{4} \div \left(\frac{5+4}{8} \right) \\ &= \frac{3}{4} \times \frac{8}{9} = \frac{2}{3} \end{aligned}$$

14. $\frac{3}{4} \times \left(\frac{5}{8} \div \frac{1}{2} \right) =$ _____.

- a) $\frac{5}{8}$ b) $\frac{2}{3}$ c) $\frac{15}{32}$ d) $\frac{15}{16}$

Solution:

Ans: d) $\frac{15}{16}$

$$\frac{3}{4} \times \frac{5}{8} \times \frac{2}{1} = \frac{15}{16}$$

15. Which of these rational numbers which have additive inverse?

- a) 7 b) $\frac{-5}{7}$
c) 0 d) All of these

Ans: (d) All of these

TRY THESE:

1. The closure property on integers holds for subtraction and not for division. What about rational numbers? Verify.

Solution:

The closure property on rational numbers also holds for subtraction and not for division.

- (i) **Closure property for subtraction:**

$$\text{Take } a = \frac{1}{2}, b = \frac{3}{4}$$

$$\text{Now, } a - b = \frac{1}{2} - \frac{3}{4} = \frac{2-3}{4} = \frac{-1}{4} \text{ is in } \mathbb{Q}$$

The rational numbers are closed under subtraction.

- (ii) **Closure property for division.**

$$\begin{aligned} a \div b &= \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} \\ &= \frac{2}{3} \text{ is in } \mathbb{Q} \end{aligned}$$

But we know that any rational number as a , $a \div 0$ is not defined.

$\therefore$ The rational numbers are not closed under division.

$$2. \text{ i) Is } \frac{3}{5} - \frac{7}{8} = \frac{7}{8} - \frac{3}{5}?$$

$$\text{ii) Is } \frac{3}{5} \div \frac{7}{8} = \frac{7}{8} \div \frac{3}{5}?$$

So, what do you conclude?

Solution:

$$(i) \quad \frac{3}{5} - \frac{7}{8} = \frac{24-35}{40} = \frac{-11}{40}$$

$$\frac{7}{8} - \frac{3}{5} = \frac{35-24}{40} = \frac{11}{40}$$

$$\frac{3}{5} - \frac{7}{8} \neq \frac{7}{8} - \frac{3}{5}$$

$$(ii) \quad \frac{3}{5} \div \frac{7}{8} = \frac{3}{5} \times \frac{8}{7} = \frac{24}{35}$$

$$\frac{7}{8} \div \frac{3}{5} = \frac{7}{8} \times \frac{5}{3} = \frac{35}{24}$$

$$\frac{3}{5} \div \frac{7}{8} \neq \frac{7}{8} \div \frac{3}{5}$$

From (i) and (ii), we conclude that, the commutative property is not true, under subtraction and division for rational numbers.

EXERCISE 1.3

1. Verify the closure property for addition and multiplication for the rational numbers $-\frac{5}{7}$ and $\frac{8}{9}$.

Solution:

$$\text{Let } a = -\frac{5}{7} \text{ and } b = \frac{8}{9}$$

- (i) **Closure property for addition:**

$$\begin{aligned} a + b &= -\frac{5}{7} + \frac{8}{9} = \frac{-45+56}{63} \\ &= \frac{11}{63} \text{ is in } \mathbb{Q}. \end{aligned}$$

$$\text{ie) } a + b \text{ is in } \mathbb{Q}$$

- (ii) **Closure property for multiplication:**

$$\begin{aligned} a \times b &= -\frac{5}{7} \times \frac{8}{9} \\ &= \frac{-40}{63} \text{ is in } \mathbb{Q}. \end{aligned}$$

$$a \times b \text{ is in } \mathbb{Q}$$

Thus a, b are in $\mathbb{Q}$, $a + b$ and $a \times b$ are also in $\mathbb{Q}$.

2. Verify the commutative property for addition and multiplication for the

rational numbers $\frac{-10}{11}$ and $\frac{-8}{33}$

Solution:

$$\text{Let } a = \frac{-10}{11} \text{ and } b = \frac{-8}{33}.$$

(i) Commutative property for addition:

$$\begin{aligned} a + b &= \frac{-10}{11} + \frac{-8}{33} = \frac{-30-8}{33} \\ &= \frac{-38}{33} \end{aligned} \quad \dots\dots\dots(1)$$

$$\begin{aligned} b + a &= \frac{-8}{33} + \frac{-10}{11} = \frac{-8-30}{33} \\ &= \frac{-38}{33} \end{aligned} \quad \dots\dots\dots(2)$$

From (1) and (2), $a + b = b + a$. Thus addition is commutative.

(ii) Commutative property for multiplication:

$$a \times b = \frac{-10}{11} \times \frac{-8}{33} = \frac{80}{363} \quad \dots\dots\dots(3)$$

$$b \times a = \frac{-8}{33} \times \frac{-10}{11} = \frac{80}{363} \quad \dots\dots\dots(4)$$

From (3) and (4), $a \times b = b \times a$. Thus multiplication is commutative.

3. Verify the associative property for addition and multiplication for the

rational numbers $\frac{-7}{9}$, $\frac{5}{6}$ and $\frac{-4}{3}$

Solution:

$$\text{Let } a = \frac{-7}{9}, b = \frac{5}{6}, c = \frac{-4}{3}$$

(i) Associative property for addition:

$$(a + b) + c = a + (b + c)$$

$$a + b = \frac{-7}{9} + \frac{5}{6}$$

$$= \frac{-14+15}{18} = \frac{1}{18}$$

$$(a + b) + c = \frac{1}{18} + \frac{-4}{3}$$

$$= \frac{1-24}{18}$$

$$= \frac{-23}{18} \quad \dots\dots\dots(1)$$

$$b + c = \frac{5}{6} + \frac{-4}{3}$$

$$= \frac{5-8}{6}$$

$$= \frac{-3}{6}$$

$$= \frac{-1}{2}$$

$$a + (b + c) = \frac{-7}{9} + \frac{-1}{2}$$

$$= \frac{-14-9}{18}$$

$$= \frac{-23}{18} \quad \dots\dots\dots(2)$$

From (1) and (2)

$$(a + b) + c = a + (b + c).$$

Thus proved.

(ii) Associative property for multiplication:

$$(a \times b) \times c = a \times (b \times c)$$

$$a \times b = \frac{-7}{9} \times \frac{5}{6} = \frac{-35}{54}$$

$$(a \times b) \times c = \frac{-35}{54} \times \frac{-4}{3}$$

$$= \frac{140}{162} = \frac{70}{81} \quad \dots\dots\dots(3)$$

$$b \times c = \frac{5}{6} \times \frac{-4}{3}$$

$$= \frac{5}{3} \times \frac{-2}{3} = \frac{-10}{9}$$

$$\begin{aligned}
 a \times (b \times c) &= \frac{-7}{9} \times \frac{-10}{9} \\
 &= \frac{70}{81} \quad \dots\dots\dots(4)
 \end{aligned}$$

From (3) and (4)

$(a \times b) \times c = a \times (b \times c)$. Thus proved.

4. Verify the distributive property

$a \times (b + c) = (a \times b) + (a \times c)$ for the rational numbers $a = \frac{-1}{2}$, $b = \frac{2}{3}$ and $c = \frac{-5}{6}$

Solution:

$$\begin{aligned}
 a &= \frac{-1}{2}, b = \frac{2}{3}, c = \frac{-5}{6} \\
 a \times (b + c) &= \frac{-1}{2} \times \left(\frac{2}{3} + \frac{-5}{6} \right) \\
 &= \frac{-1}{2} \times \left(\frac{4-5}{6} \right) \\
 &= \frac{-1}{2} \times \frac{-1}{6} \\
 &= \frac{1}{12} \quad \dots\dots\dots(1)
 \end{aligned}$$

$$\begin{aligned}
 (a \times b) + (a \times c) &= \left(\frac{-1}{2} \times \frac{2}{3} \right) + \left(\frac{-1}{2} \times \frac{-5}{6} \right) \\
 &= \frac{-2}{6} + \frac{5}{12} \\
 &= \frac{-4+5}{12} \\
 &= \frac{1}{12} \quad \dots\dots\dots(2)
 \end{aligned}$$

From (1) and (2),

$$a \times (b + c) = (a \times b) + (a \times c)$$

5. Verify the identity property for addition and multiplication for the rational numbers $\frac{15}{19}$ and $\frac{-18}{25}$.

Solution:

i) $\frac{15}{19}$

Identity property for addition:

$$\frac{15}{19} + 0 = \frac{15}{19} = 0 + \frac{15}{19}$$

Identity property for multiplication:

$$\frac{15}{19} \times 1 = \frac{15}{19} = 1 \times \frac{15}{19}$$

ii) $\frac{-18}{25}$

Identity property for addition.

$$\frac{-18}{25} + 0 = \frac{-18}{25} = 0 + \frac{-18}{25}$$

Identity property for multiplication.

$$\frac{-18}{25} \times 1 = \frac{-18}{25} = 1 \times \frac{-18}{25}$$

6. Verify the additive and multiplicative inverse property for the rational numbers $\frac{-7}{17}$ and $\frac{-17}{27}$.

i) $\frac{-7}{17}$

Additive inverse property:

$$a = \frac{-7}{17}; -a = -\left(\frac{-7}{17}\right) = \frac{7}{17}$$

$$a + (-a) = \frac{-7}{17} + \frac{7}{17} = \frac{-7+7}{17} = 0$$

$$(-a) + a = \frac{7}{17} + \frac{-7}{17} = \frac{7-7}{17} = 0$$

Multiplicative inverse property:

$$a = \frac{-7}{17}; \frac{1}{a} = \frac{17}{-7} = \frac{-17}{7}$$

$$a \times \frac{1}{a} = \frac{-7}{17} \times \frac{-17}{7} = 1$$

$$\frac{1}{a} \times a = \frac{-17}{7} \times \frac{-7}{17} = 1$$

i) $\frac{-17}{27}$

Additive inverse property:

$$a = \frac{-17}{27}; -a = -\left(\frac{-17}{27}\right) = \frac{17}{27}$$

$$a + (-a) = \frac{-17}{27} + \frac{17}{27} = \frac{-17 + 17}{27} = 0$$

$$(-a) + a = \frac{17}{27} + \frac{-17}{27} = \frac{17 - 17}{27} = 0$$

Multiplicative inverse property:

$$a = \frac{-17}{27}; \frac{1}{a} = \frac{27}{-17} = \frac{-27}{17}$$

$$a \times \frac{1}{a} = \frac{-17}{27} \times \frac{-27}{17} = 1$$

$$\frac{1}{a} \times a = \frac{-27}{17} \times \frac{-17}{27} = 1$$

OBJECTIVE TYPE QUESTIONS

7. Closure property is not true for division of rational numbers because of the number

- a) 1 b) -1
c) 0 d) $\frac{1}{2}$

Ans: c) 0

Solution:

Any number divided by 0 is not defined.
∴ Closure property is not true for division of rational number '0'.

8. $\frac{1}{2} - \left(\frac{3}{4} - \frac{5}{6}\right) \neq \left(\frac{1}{2} - \frac{3}{4}\right) - \frac{5}{6}$ illustrates that subtraction does not satisfy the _____ property for rational numbers.

- a) commutative b) closure
c) distributive d) associative

Ans: (d) associative

9. Which of the following illustrates the inverse property for addition?

- a) $\frac{1}{8} - \frac{1}{8} = 0$ b) $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

c) $\frac{1}{8} + 0 = \frac{1}{8}$ d) $\frac{1}{8} - 0 = \frac{1}{8}$

Ans: a) $\frac{1}{8} - \frac{1}{8} = 0$

Solution:

$$a + (-a) = 0$$

10. $\frac{3}{4} \times \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{3}{4} \times \frac{1}{2} - \frac{3}{4} \times \frac{1}{4}$

illustrates that multiplication is distributive over

- a) addition b) subtraction
c) multiplication d) division

Ans: b) subtraction

THINK:

1. Is the square of a prime number, prime?

The square of a prime number is not a prime. Because a prime number is a number which can be divided by 1 and by itself only. But the square of a prime number can be divided by that prime number itself.

2. Will the sum of two perfect squares always be a perfect square? What about their difference and their product?

- ♦ The sum of two perfect square may not be always a perfect square.
- ♦ Their difference also may not be a perfect square. But their product will always be a perfect square.

TRY THESE:

1. Which among 256, 576, 960, 1025, 4096 are perfect square numbers?

(Hint: Try to extend the table of squares already seen).

Solution:

No.	Its square	No.	Its square	No.	Its square
21	441	26	676	31	961

22	484	27	729	32	1024
23	529	28	784	33	1089
24	576	29	841	34	1156
25	625	30	900	35	1225
16	256	64	4096		

From the above table

256, 576 and 4096 are perfect square; 960 and 1025 are not perfect square.

2. One can judge just by look that each of the following numbers 82, 113, 1972, 2057, 8888, 24353 is not a perfect square. Explain why?

82, 113, 2057, 24353, 8888, 1972. The unit digit of these numbers are 2, 3, 7 and 8. We know that unit digit ends with 2, 3, 7 and 8 are not perfect square.

WORKED EXAMPLES:

- 1.22 The area of a square field is 3136 m^2 . Find its perimeter.

Solution:

3136 m^2

Given that the area of the square field = 3136 m^2

$\therefore$ The side of square field = $\sqrt{3136} = 56 \text{ m}$

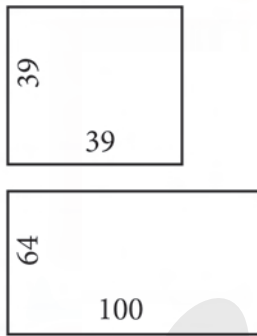
$$\begin{array}{r} 56 \\ 5 \overline{) 3136} \\ \underline{25} \\ 636 \\ 106 \overline{) 636} \\ \underline{636} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \text{The perimeter of the square field} &= 4 \times \text{side} \\ &= 4 \times 56 \\ &= 224 \text{ m} \end{aligned}$$

- 1.23 A real estate owner had two plots, a square plot of side 39 m and a rectangular plot of dimensions 100 m length and 64 m breadth. He sells both of these plots and acquires a new square plot of the same area. What is the length of side of his new plot?

Solution:

The transactions can be visualised as follows:

Plots sold	New Plot bought
	

$$\begin{aligned} \text{Area of the square plot bought} &= \text{Area of the square plot sold} + \text{Area of the rectangular plot sold} \\ &= 39 \times 39 + 100 \times 64 \\ &= 1521 + 6400 \\ &= 7921 \text{ m}^2 \end{aligned}$$

Length of a side of the new square plot:

$$= \sqrt{7921} = 89 \text{ m}$$

$$\begin{array}{r} 89 \\ 8 \overline{) 7921} \\ \underline{64} \\ 1521 \\ 169 \overline{) 1521} \\ \underline{1521} \\ 0 \end{array}$$

- 1.24 Find the square root of 324 by prime factorisation.

Solution:

First, resolve the given number into prime factors. Group the identical factors in pairs and then take one from them to find the square root.

NUMBERS

Now, $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$$= 2^2 \times 3^2 \times 3^2$$

$$= (2 \times 3 \times 3)^2$$

$$\therefore \sqrt{324} = \sqrt{(2 \times 3 \times 3)^2}$$

$$= 2 \times 3 \times 3 = 18$$

2	324
2	162
3	81
3	27
3	9
3	3
	1

- 1.25 Find the least number by which 250 is to be multiplied (or) divided so that the resulting number is a perfect square. Also, find the square root in that case.**

Solution:

Here, $250 = 5 \times 5 \times 5 \times 2$

$$= 5^2 \times 5 \times 2$$

Here, the prime factors 5 and 2 do not have pairs.

Therefore, we can either divide 250 by 10 (5×2) or multiply 250 by 10.

(i) If we multiply 250 by 10,

we get $2500 = 5^2 \times 5 \times 2 \times 5 \times 2$ and therefore the square root of 2500 would be $5 \times 5 \times 2 = 50$.

(ii) If we divide 250 by 10, we get 25 and in this case we get $\sqrt{25} = \sqrt{5^2} = 5$.

5	250
5	50
5	10
2	2
	1

- 1.26 Is 108 a perfect square number?**

Solution:

Here, $108 = 2 \times 2 \times 3 \times 3 \times 3$

$$= 2^2 \times 3^2 \times 3$$

Here, the prime factor 3 does not have a second pair. Hence, 108 is not a perfect square number.

2	108
2	54
3	27
3	9
3	3
	1

- 1.27 Find the square root of 459684 by long division method.**

Solution:

By long division method, we can find the square root of 459684 as given below:



Don

		6	7	8
×2	6	45	96	84
		36	↓	
→	12	9	96	
×2		8	89	↓
→	1348	1	07	84
		1	07	84
			0	

$$\sqrt{459684} = 678$$

TRY THESE:

1. Find the square root by long division method.

(i) 400

(ii) 1764

(iii) 9801

Solution:

i) 400

	20
2	4 00
	4
	00

$$\therefore \sqrt{400} = 20$$

ii) 1764

	42
4	17 64
	16
82	164 164
	0

$$\therefore \sqrt{1764} = 42$$

iii) 9801

	99
9	<u>98</u> <u>01</u>
	81
189	1701
	1701
	0

$$\therefore \sqrt{9801} = 99$$

2. Without calculating the square root, guess the number of digits in the square root of the following numbers:

(i) 14400

(ii) 390625

(iii) 100000000

Solution:

(i) 14400

The number of digits in 14400 is 5.

 $\therefore$ The number of digits in the square root

$$\text{of } 14400 = \frac{n+1}{2} = \frac{5+1}{2} \\ = 3$$

(ii) 390625

The number of digits in 390625 is 6.

 $\therefore$ The number of digits in the square

$$\text{root of } 390625 = \frac{n}{2} = \frac{6}{2} \\ = 3$$

(iii) 100000000

The number of digits in 100000000 is 9.

 $\therefore$ The number of digits in the square

$$\text{root of } 100000000 = \frac{n+1}{2} = \frac{9+1}{2} \\ = 5$$

3. Find the square root of

(i) 5.4756 (ii) 19.36 (iii) 116.64

Solution:

i) 5.4756

$$\begin{array}{r} 2.34 \\ 2 \overline{) 5.47 \overline{) 56}} \\ \underline{4} \\ 147 \\ 43 \overline{) 147} \\ \underline{129} \\ 1856 \\ 464 \overline{) 1856} \\ \underline{1856} \\ 0 \end{array}$$

$$\sqrt{5.4756} = 2.34$$

ii) 19.36

$$\begin{array}{r} 4.4 \\ 4 \overline{) 19.36} \\ \underline{16} \\ 336 \\ 84 \overline{) 336} \\ \underline{336} \\ 0 \end{array}$$

$$\sqrt{19.36} = 4.4$$

iii) 116.64

$$\begin{array}{r} 10.8 \\ 1 \overline{) 116.64} \\ \underline{1} \\ 1664 \\ 208 \overline{) 1664} \\ \underline{1664} \\ 0 \end{array}$$

$$\therefore \sqrt{116.64} = 10.8$$

1.28 Find the value of $\sqrt{256}$ *Solution:*

$$\sqrt{256} = \sqrt{16 \times 16} = \sqrt{16} \times \sqrt{16} = 4 \times 4 = 16$$

$$(\text{or}) \sqrt{256} = \sqrt{64 \times 4} = \sqrt{64} \times \sqrt{4} = 8 \times 2 = 16$$

1.29 Find the value of $\sqrt{42.25}$ *Solution:*

We can write this as

$$\sqrt{42.25} = \sqrt{\frac{4225}{100}} = \frac{\sqrt{4225}}{\sqrt{100}}$$

Now, it is easy to compute the square root of the whole number 4225 by long division method as $\sqrt{4225} = 65$ and so, we now get

$$\sqrt{42.25} = \sqrt{\frac{4225}{100}} = \frac{\sqrt{4225}}{\sqrt{100}} = \frac{65}{10} = 6.5$$

TRY THESE:

Find the square root of 1.2321 and 11.9025 by using the formula $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Solution:

$$(i) \sqrt{1.2321} = \sqrt{\frac{12321}{10000}} = \frac{\sqrt{12321}}{\sqrt{10000}}$$

$$\begin{array}{r} 111 \\ 1 \overline{) 12321} \\ \underline{1} \\ 23 \\ 21 \\ \underline{221} \\ 221 \\ \underline{0} \end{array} \quad \begin{array}{r} 100 \\ 1 \overline{) 10000} \\ \underline{1} \\ 0 \end{array}$$

$$\frac{\sqrt{12321}}{\sqrt{10000}} = \frac{111}{100}$$

$$= 1.11$$

$$(ii) \sqrt{11.9025} = \sqrt{\frac{119025}{10000}} = \frac{\sqrt{119025}}{\sqrt{10000}}$$

$$\begin{array}{r} 345 \\ 3 \overline{) 119025} \\ \underline{9} \\ 290 \\ 256 \\ \underline{3425} \\ 3425 \\ \underline{0} \end{array} \quad \begin{array}{r} 100 \\ 1 \overline{) 10000} \\ \underline{1} \\ 0 \end{array}$$

$$\frac{\sqrt{119025}}{\sqrt{10000}} = \frac{345}{100} = 3.45$$

1.30 Simplify: (i) $\sqrt{12} \times \sqrt{3}$ (ii) $\sqrt{\frac{98}{162}}$

Solution:

(i) Remembering the rule, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$$\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = \sqrt{6^2} = 6$$

(ii) Remembering the rule, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ ($b \neq 0$)

$$\sqrt{\frac{98}{162}} = \sqrt{\frac{2 \times 49}{2 \times 81}} = \sqrt{\frac{49}{81}} = \sqrt{\frac{7^2}{9^2}} = \frac{7}{9}$$

1.31 Simplify: (i) $\sqrt{2\frac{7}{9}}$ (ii) $\sqrt{1\frac{9}{16}}$

Solution:

$$i) \sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \sqrt{\frac{5^2}{3^2}} = \frac{5}{3} = 1\frac{2}{3}$$

$$ii) \sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \sqrt{\frac{5^2}{4^2}} = \frac{5}{4} = 1\frac{1}{4}$$

TRY THESE:

Write the numbers in ascending order

(i) 4, $\sqrt{14}$, 5 (ii) 7, $\sqrt{65}$, 8

Solution:

(i) 4, $\sqrt{14}$, 5

Here $\sqrt{14}$ is not a square number. We know that the two closest squares surrounding 14 are 9 and 16.

Thus $9 < 14 < 16$ which can be written as $3^2 < \sqrt{14} < 4^2$

$\therefore$ The ascending order of 4, $\sqrt{14}$, 5 is $\sqrt{14} < 4 < 5$

(ii) 7, $\sqrt{65}$, 8

Here $\sqrt{65}$ is not a square number. We know that the two closest squares surrounding 65 are 64 and 81.

Thus $64 < 65 < 81$ which can be written as $8^2 < \sqrt{65} < 9^2$

$\therefore$ The ascending order of 7, $\sqrt{65}$, 8 is 7, 8, $\sqrt{65}$.

EXERCISE 1.4

1. Fill in the blanks:

- (i) The ones digit in the square of 77 is ____.

Ans: 9

Solution: $77^2 = 77 \times 77 = 5929$

- (ii) The number of non-square numbers between
- 24^2
- and
- 25^2
- is ____.

Ans: 48

Solution:

"We know that, Between the squares of the consecutive numbers n and $(n+1)$, there are $2n$ non-square numbers".

$$n = 24 \text{ and } n + 1 = 25$$

$$\therefore 2n = 2 \times 24 = 48$$

- (iii) The number of perfect square numbers between 300 and 500 is ____.

Ans: 5

Solution:

(i) $18^2 = 324$

(ii) $19^2 = 361$

(iii) $20^2 = 400$

(iv) $21^2 = 441$

(v) $22^2 = 484$

- iv) If a number has 5 or 6 digits in it, then its square root will have ____ digits.

Ans: 3

Solution:

If n is the number of digits in a number then, the number of digits in the square

$$\text{root of that number is} = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

$$= \begin{cases} \frac{5+1}{2} & \text{if } n \text{ is odd} \\ \frac{6}{2} & \text{if } n \text{ is even} \end{cases} = 3$$

- v) The value of
- $\sqrt{180}$
- lies between integers ____ and ____.

Ans: 13, 14

Solution:

$$\begin{array}{r} 13.4 \\ 1 \overline{) 180.00} \\ \underline{1} \\ 23 \\ \underline{80} \\ 69 \\ 264 \overline{) 1100} \\ \underline{1056} \\ 44 \end{array}$$

 $\sqrt{180} = 13.4$ which lies between 13 and 14.

2. Say True or False:

- (i) When a square number ends in 6, its square root will have 6 in the unit's place.

Ans: True.

Reason: $\sqrt{36} = 6$; $\sqrt{256} = 16$ etc.

- (ii) A square number will not have odd number of zeros at the end.

Ans: True.

Reason:

100 and 10,000 are square numbers, they have even number of zeros.

- (iii) The number of zeros in the square of 91000 is 9.

Ans: False.

Reason: $(91000)^2 = 8281000000$.

The number of zeros in the square of 91000 is 6.

- (iv) The square of 75 is 4925.

Ans: False.

- (v) The square root of 225 is 15.

Ans: True.

3. Find the square of the following numbers.

- (i) 17 (ii) 203 (iii) 1098

Solution:

$$(i) 17^2 = 17 \times 17 = 289$$

$$(ii) 203^2 = 203 \times 203 = 41209$$

$$(iii) 1098^2 = 1098 \times 1098 = 1205604$$

4. Examine if each of the following is a perfect square:

- (i) 725 (ii) 190 (iii) 841 (iv) 1089

Solution:

$$(i) 725 = 5 \times 5 \times 29$$

5	725
5	145
	29

The prime factor 29 does not have a pair.

Hence 725 is not a perfect square number.

$$(ii) 190 = 2 \times 5 \times 19$$

2	190
5	95
	19

Here all the three prime factors do not have pairs.

Hence 190 is not a perfect square number.

$$(iii) 841 = 29 \times 29$$

29	841
	29

Here the prime factor 29 has pair.

Hence 841 is a perfect square number.

$$(iv) 1089 = 3 \times 3 \times 11 \times 11$$

3	1089
3	363
11	121
	11

Here all the prime factors have pairs

Hence 1089 is a perfect square number.

5. Find the square root by prime factorisation method.

- (i) 144 (ii) 256 (iii) 784
(iv) 1156 (v) 4761 (vi) 9025

Solution:

- (i) 144

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$= 2^2 \times 2^2 \times 3^2$$

$$= (2 \times 2 \times 3)^2$$

$$\sqrt{144} = \sqrt{(2 \times 2 \times 3)^2}$$

$$= 2 \times 2 \times 3 = 12$$

2	144
2	72
2	36
2	18
3	9
3	3
	1

- (ii) 256

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^2 \times 2^2 \times 2^2 \times 2^2$$

$$= (2 \times 2 \times 2 \times 2)^2$$

$$\sqrt{256} = \sqrt{(2 \times 2 \times 2 \times 2)^2}$$

$$= 2 \times 2 \times 2 \times 2$$

$$= 16$$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

- (iii) 784

$$784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$$

$$= 2^2 \times 2^2 \times 7^2$$

$$= (2 \times 2 \times 7)^2$$

$$\sqrt{784} = \sqrt{(2 \times 2 \times 7)^2}$$

$$= 2 \times 2 \times 7$$

$$= 28$$

2	784
2	392
2	196
2	98
7	49
	7

- (iv) 1156

$$= 2 \times 2 \times 17 \times 17$$

$$= 2^2 \times 17^2$$

$$= (2 \times 17)^2$$

$$\therefore \sqrt{1156} = \sqrt{(2 \times 17)^2}$$

$$= 2 \times 17$$

$$= 34$$

2	1156
2	578
17	289
	17

(v) $4761 = 3 \times 3 \times 23 \times 23$

$$= 3^2 \times 23^2$$

$$= (3 \times 23)^2$$

$$\sqrt{4761} = \sqrt{(3 \times 23)^2}$$

$$= 3 \times 23$$

$$= 69$$

3	4761
3	1587
23	529
	23

(vi) $9025 = 5 \times 5 \times 19 \times 19$

$$= 5^2 \times 19^2$$

$$= (5 \times 19)^2$$

$$\sqrt{9025} = \sqrt{(5 \times 19)^2}$$

$$= 5 \times 19$$

$$= 95$$

5	9025
5	1805
19	361
	19

iii) $\sqrt{11025}$

	105
1	110 25
	1
205	1025
	1025
	0

$$\therefore \sqrt{11025} = 105$$

iv) $\sqrt{17956}$

	134
1	179 56
	1
23	79
	69
264	1056
	1056
	0

$$\therefore \sqrt{17956} = 134$$

v) $\sqrt{418609}$

	647
6	41 86 09
	36
124	586
	496
1287	9009
	9009
	0

$$\therefore \sqrt{418609} = 647$$

6. Find the square root by long division method.

(i) 1764

(ii) 6889

(iii) 11025

(iv) 17956

(v) 418609

Solution:

i) $\sqrt{1764}$

	42
4	17 64
	16
82	164
	164
	0

$$\therefore \sqrt{1764} = 42$$

ii) $\sqrt{6889}$

	83
8	68 89
	64
163	489
	489
	0

$$\therefore \sqrt{6889} = 83$$

7. Estimate the value of the following square roots to the nearest whole number:

i) $\sqrt{440}$ ii) $\sqrt{800}$ iii) $\sqrt{1020}$

Solution:

i) $\sqrt{440} = 20.9$

$$\begin{array}{r} 20.9 \\ 2 \overline{) 440} \\ \underline{4} \\ 409 \\ \underline{400} \\ 3681 \\ \underline{3600} \\ 319 \end{array}$$

The nearest whole number of $\sqrt{440}$ is 21.

ii) $\sqrt{800} = 28.2$

$$\begin{array}{r} 28.2 \\ 2 \overline{) 800.00} \\ \underline{4} \\ 48 \\ \underline{400} \\ 384 \\ 562 \\ \underline{1600} \\ 1124 \\ \underline{1120} \\ 576 \end{array}$$

The nearest whole number of $\sqrt{800}$ is 28.

iii) $\sqrt{1020} = 31.9$

$$\begin{array}{r} 31.9 \\ 3 \overline{) 1020.00} \\ \underline{9} \\ 61 \\ \underline{120} \\ 61 \\ 629 \\ \underline{5900} \\ 5661 \\ \underline{5670} \\ 339 \end{array}$$

$\therefore$ The nearest whole number of $\sqrt{1020} = 32$

8. Find the square root of the following decimal numbers and fractions:

i) 2.89 ii) 67.24

iii) 2.0164 iv) $\frac{144}{225}$ v) $7\frac{18}{49}$

Solution:

(i) $\sqrt{2.89}$

$$\begin{array}{r} 1.7 \\ 1 \overline{) 2.89} \\ \underline{1} \\ 27 \\ \underline{189} \\ 189 \\ \underline{189} \\ 0 \end{array}$$

$\sqrt{2.89} = 1.7$

(ii) $\sqrt{67.24}$

$$\begin{array}{r} 8.2 \\ 8 \overline{) 67.24} \\ \underline{64} \\ 162 \\ \underline{128} \\ 324 \\ \underline{324} \\ 0 \end{array}$$

$\sqrt{67.24} = 8.2$

(iii) $\sqrt{2.0164}$

$$\begin{array}{r} 1.42 \\ 1 \overline{) 2.0164} \\ \underline{1} \\ 24 \\ \underline{101} \\ 96 \\ 282 \\ \underline{564} \\ 564 \\ \underline{564} \\ 0 \end{array}$$

$\sqrt{2.0164} = 1.42$

$$(iv) \sqrt{\frac{144}{225}} = \frac{\sqrt{144}}{\sqrt{225}}$$

$$\begin{array}{r} 12 \\ 1 \overline{) 144} \\ \underline{1} \\ 22 \overline{) 44} \\ \underline{44} \\ 0 \end{array} \quad \begin{array}{r} 15 \\ 1 \overline{) 225} \\ \underline{1} \\ 25 \overline{) 125} \\ \underline{125} \\ 0 \end{array}$$

$$= \frac{\sqrt{12 \times 12}}{\sqrt{15 \times 15}} = \frac{12}{15}$$

$$v) \sqrt{7 \frac{18}{49}} = \sqrt{\frac{361}{49}} = \frac{\sqrt{361}}{\sqrt{49}}$$

$$\begin{array}{r} 19 \\ 1 \overline{) 361} \\ \underline{1} \\ 29 \overline{) 261} \\ \underline{261} \\ 0 \end{array} \quad \begin{array}{r} 7 \\ 7 \overline{) 49} \\ \underline{49} \\ 0 \end{array}$$

$$= \frac{19}{7} = 2 \frac{5}{7}$$

9. Find the least number that must be subtracted to 6666 so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

Solution:

$$\begin{array}{r} 81 \\ 8 \overline{) 6666} \\ \underline{64} \\ 161 \overline{) 266} \\ \underline{161} \\ 105 \end{array}$$

The remainder is 105. 105 is subtracted from 6666 so that it becomes a perfect square. The square root of perfect square is 81.

10. Find the least number by which 1800 should be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

Solution:

$$1800 = 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 3$$

To get a perfect square 2 should be multiplied with 1800.

$$\text{i.e. } 1800 \times 2 = 3600$$

$$= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 3$$

$$= 2^2 \times 2^2 \times 5^2 \times 3^2$$

$$= (2 \times 2 \times 5 \times 3)^2$$

$$\therefore \sqrt{3600} = \sqrt{(2 \times 2 \times 5 \times 3)^2}$$

$$= 2 \times 2 \times 5 \times 3$$

$$= 60$$

$$\begin{array}{r} 2 \overline{) 1800} \\ \underline{2} \\ 2 \overline{) 900} \\ \underline{2} \\ 5 \overline{) 225} \\ \underline{5} \\ 5 \overline{) 45} \\ \underline{3} \\ 3 \overline{) 9} \\ \underline{3} \\ 3 \overline{) 3} \\ \underline{3} \\ 1 \end{array}$$

OBJECTIVE TYPE QUESTIONS

11. The square of 43 ends with the digit ____.

a) 9 b) 6 c) 4 d) 3

Ans: a) 9

12. ____ is added to 24^2 to get 25^2 .

a) 4^2 b) 5^2 c) 6^2 d) 7^2

Ans: d) 7^2

Solution:

$$24^2 = 576;$$

$$25^2 = 625; 625 - 576 = 49 = 7^2$$

13. $\sqrt{48}$ is approximately equal to

a) 5 b) 6 c) 7 d) 8

Ans: c) 7

Solution:

$$\sqrt{48} = 6.9$$

$$\begin{array}{r} 6.9 \\ 6 \overline{) 48.00} \\ \underline{36} \\ 129 \overline{) 1200} \\ \underline{1161} \\ 39 \end{array}$$

$$\therefore \sqrt{48} \text{ is approximately equal to } 7$$

14. $\sqrt{128} - \sqrt{98} + \sqrt{18} =$

- a) $\sqrt{2}$ b) $\sqrt{8}$ c) $\sqrt{48}$ d) $\sqrt{32}$

Ans: d) $\sqrt{32}$

Solution:

$$\begin{aligned}\sqrt{128} &= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = 8\sqrt{2} \\ \sqrt{98} &= \sqrt{2 \times 7 \times 7} = 7\sqrt{2} \\ \sqrt{18} &= \sqrt{2 \times 3 \times 3} = 3\sqrt{2} \\ \therefore \sqrt{128} - \sqrt{98} + \sqrt{18} \\ &= 8\sqrt{2} - 7\sqrt{2} + 3\sqrt{2} \\ &= 4\sqrt{2} = \sqrt{4 \times 4 \times 2} \\ &= \sqrt{32} \\ \sqrt{128} - \sqrt{98} + \sqrt{18} &= \sqrt{32}\end{aligned}$$

15. The number of digits in the square root of 123454321 is

- a) 4 b) 5 c) 6 d) 7

Ans: b) 5

Solution:

The number 123454321 has 9 digits

$$\begin{aligned}\text{The number of digits in } \sqrt{123454321} \\ &= \frac{9+1}{2} = 5\end{aligned}$$

TRY THESE:

1. Find the ones digit in the cubes of each of the following numbers.

- (i) 12 (ii) 27 (iii) 38
(iv) 53 (v) 71 (vi) 84

Solution:

(i) 12

If a natural number ends with 2 its cube ends with 8.

$$12^3 = 12 \times 12 \times 12 = 1728$$

(ii) 27

If a natural number ends with 7 its cube ends with 3

$$27^3 = 27 \times 27 \times 27 = 19683$$

(iii) 38

If a natural number ends with 8 its cube ends with 2.

$$38^3 = 38 \times 38 \times 38 = 4972$$

(iv) 53

If a natural number ends with 3 its cube ends with 7.

$$53^3 = 53 \times 53 \times 53 = 148877$$

(v) 71

If a natural number ends with 1 its cube ends with 1.

$$71^3 = 71 \times 71 \times 71 = 357911$$

(vi) 84

If a natural number ends with 4 its cube ends with 4.

$$\begin{aligned}84^3 &= 84 \times 84 \times 84 \\ &= 592704\end{aligned}$$

WORKED EXAMPLES

1.32 Is 400 a perfect cube?

Solution:

By prime factorisation, we have

$$400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$$

There is only one triplet. To make further triplets, we will need two more 2's and one more 5. Therefore, 400 is not a perfect cube.

1.33 Find the smallest number by which 675 must be multiplied to obtain a perfect cube.

Solution:

We find that,

$$675 = 3 \times 3 \times 3 \times 5 \times 5 \dots (1)$$

Grouping the prime factors of 675 as

triplets, we are left over with 5×5 .

We need one more 5 to make it a perfect cube.

To make 675 a perfect cube, multiply both sides of (1) by 5.

$$675 \times 5 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

Now, 3375 is a perfect cube. Thus, the smallest required number to multiply 675 such that the new number perfect cube is 5.

3	675
3	225
3	75
5	25
5	5
	1

1.34 Find the cube root of 27000.

Solution:

By prime factorisation, we have

$$27000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$\therefore \sqrt[3]{27000} = 2 \times 3 \times 5 = 30$$

1.35 Evaluate: i) $\sqrt[3]{\frac{9261}{8000}}$ ii) $\sqrt[3]{\frac{1728}{729}}$

Solutions:

$$\begin{aligned} \text{(i)} \quad \sqrt[3]{\frac{9261}{8000}} &= \frac{\sqrt[3]{9261}}{\sqrt[3]{8000}} = \frac{(21 \times 21 \times 21)^{\frac{1}{3}}}{(20 \times 20 \times 20)^{\frac{1}{3}}} \\ &= \frac{(21^3)^{\frac{1}{3}}}{(20^3)^{\frac{1}{3}}} = \frac{21}{20} = 1\frac{1}{20} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \sqrt[3]{\frac{1728}{729}} &= \frac{\sqrt[3]{1728}}{\sqrt[3]{729}} = \frac{(12 \times 12 \times 12)^{\frac{1}{3}}}{(9 \times 9 \times 9)^{\frac{1}{3}}} \\ &= \frac{(12^3)^{\frac{1}{3}}}{(9^3)^{\frac{1}{3}}} = \frac{12}{9} = \frac{4}{3} = 1\frac{1}{3} \end{aligned}$$

EXERCISE 1.5

1. Fill in the blanks:

- (i) The ones digit in the cube of 73 is _____.
Ans: 7

Solution:

The ones digit in the cube of 73 is 3.

$$\therefore 3^3 = 27$$

- (ii) The maximum number of digits in the cube of a two digit number is _____.
Ans: 6

- (iii) The smallest number to be added to 3333 to make it a perfect cube is _____.
Ans: 42

Solution:

3375 is the perfect cube after the number 3333.

Let x be the smallest number.

$$x + 3333 = 3375$$

$$x = 3375 - 3333$$

$$x = 42$$

- (iv) The cube root of 540×50 is _____.
Ans: 30

Solution:

$$\begin{aligned} 540 \times 50 &= 5 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 2 \\ &= 5^3 \times 3^3 \times 2^3 \\ &= (5 \times 3 \times 2)^3 \end{aligned}$$

$$\begin{aligned} \sqrt[3]{540 \times 50} &= \sqrt[3]{(5 \times 3 \times 2)^3} \\ &= 5 \times 3 \times 2 \\ &= 30 \end{aligned}$$

5	540
2	108
2	54
3	27
3	9
	3

- (iv) The cube root of 0.000004913 is _____.
Ans: 0.017

Solution:

$$\begin{aligned} 0.000004913 &= \frac{4913}{1000000000} \\ &= \frac{17 \times 17 \times 17}{1000 \times 1000 \times 1000} \end{aligned}$$

$$= \frac{17^3}{1000^3} = \left(\frac{17}{1000}\right)^3$$

$$\therefore \sqrt[3]{0.000004913} = \sqrt[3]{\left(\frac{17}{1000}\right)^3} = \frac{17}{1000}$$

$$= 0.017$$

2. Say True or False:

i) The cube of 24 ends with the digit 4.

Ans: True

ii) Subtracting 10^3 from 1729 gives 9^3 .

Ans: True

iii) The cube of 0.0012 is 0.00001728.

Ans: False

Reason:

$$0.0012 = \frac{12}{10000}$$

$$(0.0012)^3 = \left(\frac{12}{10000}\right)^3 = \frac{1728}{10000000000000}$$

$$= 0.000000001728$$

(iv) 79570 is not a perfect cube.

Ans: True

Reason:

$$79570 = 5 \times 2 \times 7957$$

5	79570
2	15914
	7957

There is no form of triplets.

(v) The cube root of 250047 is 63.

Ans: True

Solution:

$$250047 = 9 \times 9 \times 9 \times 7 \times 7 \times 7$$

$$= 9^3 \times 7^3$$

$$= (9 \times 7)^3$$

$$\therefore \sqrt[3]{250047} = \sqrt[3]{(9 \times 7)^3}$$

$$= 9 \times 7$$

$$= 63$$

9	250047
9	27783
9	3087
7	343
7	49
	7

3. Show that 1944 is not a perfect cube.

Solution:

$$1944 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$$

Here there are two triplets and we need one more 3.

Hence 1944 is not a perfect cube.

2	1944
2	972
2	486
3	243
3	81
3	27
3	9
	3

4. Find the smallest number by which 10985 should be divided so that the quotient is a perfect cube.

Solution:

$$10985 = 13 \times 13 \times 13 \times 5$$

We have one triplets.

5 is left over.

$\therefore$ The smallest number by which 10985 should be divided is 5 so that the quotient is a perfect cube.

5	10985
13	2197
13	169
	13

5. Find the smallest number by which 200 should be multiplied to make it a perfect cube.

Solution:

$$200 = 2 \times 2 \times 2 \times 5 \times 5$$

We have one triplets. We need one more 5 to get another triplet.

$\therefore$ 5 should be multiplied to make 200 a perfect cube.

6. Find the cube root of $24 \times 36 \times 80 \times 25$

Solution:

$$\sqrt[3]{24 \times 36 \times 80 \times 25}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5}$$

$$= \sqrt[3]{2^3 \times 3^3 \times 2^3 \times 2^3 \times 5^3}$$

$$= \sqrt[3]{(2 \times 3 \times 2 \times 2 \times 5)^3}$$

$$= ((2 \times 3 \times 2 \times 2 \times 5)^3)^{1/3}$$

$$= 2 \times 3 \times 2 \times 2 \times 5 = 120$$

7. Find the cube root of 729 and 6859 by prime factorisation.

Solution:

$$\begin{aligned} 729 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^3 \times 3^3 \\ \therefore 729 &= (3 \times 3)^3 \\ \sqrt[3]{729} &= \sqrt[3]{(3 \times 3)^3} \\ &= 3 \times 3 \\ &= 9 \end{aligned}$$

$$\therefore \sqrt[3]{729} = 9$$

$$\begin{aligned} 6859 &= 19 \times 19 \times 19 \\ &= 19^3 \end{aligned}$$

$$\therefore \sqrt[3]{6859} = \sqrt[3]{19^3}$$

$$\sqrt[3]{6859} = 19$$

3	729
3	243
3	81
3	27
3	9
	3

19	6859
19	361
	19

8. What is the square root of cube root of 46656?

Solution:

$$\begin{aligned} 46656 &= 9 \times 9 \times 9 \times 4 \times 4 \times 4 \\ &= 9^3 \times 4^3 \\ \sqrt[3]{46656} &= \sqrt[3]{(9 \times 4)^3} \\ &= 9 \times 4 \\ &= 36 \end{aligned}$$

$$\therefore \text{The square root of cube root of 46656 is } \sqrt{36} = 6$$

9	46656
9	5184
9	576
4	64
4	16
	4

9. If the cube of a squared number is 729, find the square root of that number.

Solution:

Let x be the cube of a squared number.
By the given data,

$$(x^2)^3 = 729$$

$$x^2 = 9$$

$$x = \sqrt{9}$$

$$x = 3$$

$$\sqrt{x} = \sqrt{3}$$

$$\therefore \text{The square root of that number is } \sqrt{3}.$$

10. Find the two smallest perfect square numbers which when multiplied together gives a perfect cube number.

Solution:

$$\underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} = 4 \times 16$$

4 and 16 are perfect square.

And also $4 \times 16 = 64$ is a perfect cube.

TRY THESE:

1. Expand the following numbers using exponents:

(i) 8120

(ii) 20,305

(iii) 3652.01

(iv) 9426.521

Solution:

$$\begin{aligned} \text{(i) } 8120 &= 8000 + 100 + 20 + 0 \\ &= (8 \times 1000) + (1 \times 100) + (2 \times 10) + (0 \times 1) \\ &= (8 \times 10^3) + (1 \times 10^2) + (2 \times 10^1) + (0 \times 10^0) \end{aligned}$$

$$\begin{aligned} \text{(ii) } 20305 &= 20000 + 300 + 5 \\ &= (2 \times 10000) + (3 \times 100) + (5 \times 1) \\ &= 2 \times 10^4 + 3 \times 10^2 + 5 \times 10^0 \end{aligned}$$

$$\begin{aligned} \text{(iii) } 3652.01 &= 3000 + 600 + 50 + 2 + \frac{0}{10} + \frac{1}{100} \\ &= (3 \times 1000) + (6 \times 100) + (5 \times 10) + (2 \times 1) \\ &\quad + (0 \times 10^{-1}) + (1 \times 10^{-2}) \\ &= (3 \times 10^3) + (6 \times 10^2) + (5 \times 10) + (2 \times 10^0) \\ &\quad + (0 \times 10^{-1}) + (1 \times 10^{-2}) \end{aligned}$$

$$\begin{aligned} \text{(iv) } 9426.521 &= 9000 + 400 + 20 + 6 + \\ &\quad \frac{5}{10} + \frac{2}{100} + \frac{1}{1000} \\ &= (9 \times 1000) + (4 \times 100) + (2 \times 10) + (6 \times 1) \\ &\quad + \frac{5}{10} + \frac{2}{100} + \frac{1}{1000} \\ &= (9 \times 10^3) + (4 \times 10^2) + (2 \times 10^1) + (6 \times 10^0) \\ &\quad + (5 \times 10^{-1}) + (2 \times 10^{-2}) + (1 \times 10^{-3}) \end{aligned}$$

2. Verify the following rules (as we did above). Here, a, b are non-zero integers and m, n are any integers.

1. Product of same powers to power of product rule: $a^m \times b^m = (ab)^m$

Solution:

$$a^m \times b^m = (ab)^m$$

Take $a = 5$, $b = 3$ and $m = 2$

$$\begin{aligned} 5^2 \times 3^2 &= 25 \times 9 \\ &= 225 \\ &= 15^2 \\ &= (5 \times 3)^2 \end{aligned}$$

2. Quotient of same powers to power of

$$\text{quotient rule: } \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Solution:

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Take $a = 4$, $b = 2$ and $m = 2$

$$\text{LHS} = \frac{a^m}{b^m} = \frac{4^2}{2^2} = \frac{16}{4} = 4$$

$$\text{RHS} = \left(\frac{a}{b}\right)^m = \left(\frac{4}{2}\right)^2 = 2^2 = 4$$

$\therefore \text{LHS} = \text{RHS}$

3. Zero exponent rule: $a^0 = 1$.

Solution:

Take $a = 10$

$$\begin{aligned} \therefore a^0 &= 10^0 = 10^{1-1} = 10^1 \times 10^{-1} \\ &= \frac{10^1}{10^1} = \frac{10}{10} = 1 \end{aligned}$$

1.36 Find the value of

$$\begin{aligned} \text{i) } 4^{-3} & \quad \text{ii) } \frac{1}{2^{-3}} \\ \text{iii) } (-2)^5 \times (-2)^{-3} & \quad \text{iv) } \frac{3^2}{3^{-2}} \end{aligned}$$

Solution:

$$\text{i) } 4^{-3} = \frac{1}{4^3} = \frac{1}{4 \times 4 \times 4} = \frac{1}{64}$$

$$\text{ii) } \frac{1}{2^{-3}} = 2^3 = 2 \times 2 \times 2 = 8$$

$$\text{iii) } (-2)^5 \times (-2)^{-3} = (-2)^{5-3} = (-2)^2 = -2 \times -2 = 4$$

$$\text{iv) } \frac{3^2}{3^{-2}} = 3^2 \times 3^2 = 9 \times 9 = 81$$

1.37 Simplify and write the answer in exponential form:

$$\text{(i) } (3^5 \div 3^8)^5 \times 3^{-5} \quad \text{(ii) } (-3)^4 \times \left(\frac{5}{3}\right)^4$$

Solution:

$$\text{(i) } \left(\frac{3^5}{3^8}\right)^5 \times 3^{-5}$$

$$\begin{aligned} &= (3^{5-8})^5 \times 3^{-5} = (3^{-3})^5 \times 3^{-5} = 3^{-3 \times 5} \times 3^{-5} = 3^{-15} \times 3^{-5} \\ &= 3^{-15-5} = 3^{-20} = \frac{1}{3^{20}} \end{aligned}$$

$$\text{(ii) } (-3)^4 \times \left(\frac{5}{3}\right)^4 = 3^4 \times \frac{5^4}{3^4} = 5^4 = 625$$

1.38 Find x so that $(-7)^{x+2} \times (-7)^5 = (-7)^{10}$

Solution:

$$(-7)^{x+2} \times (-7)^5 = (-7)^{10}$$

$$(-7)^{x+2+5} = (-7)^{10}$$

Since the bases are equal, we equate the exponents to get

$$x + 7 = 10$$

$$x = 10 - 7 = 3$$

1.39 Combine the scientific notations:

$$\text{(i) } (7 \times 10^2) (5.2 \times 10^7)$$

$$\text{(ii) } (3.7 \times 10^{-5}) (2 \times 10^{-3})$$

Solution:

$$\begin{aligned} \text{(i) } (7 \times 10^2) (5.2 \times 10^7) &= 36.4 \times 10^9 \\ &= 3.64 \times 10^{10} \end{aligned}$$

$$\text{(ii) } (3.7 \times 10^{-5}) (2 \times 10^{-3}) = 7.4 \times 10^{-8}$$

1.40 Write the following scientific notations in standard form:

$$\text{(i) } 2.27 \times 10^{-4}$$

$$\text{(ii) Light travels at } 1.86 \times 10^5 \text{ miles per second.}$$

Solution:

$$\text{(i) } 2.27 \times 10^{-4} = 0.000227$$

$$\begin{aligned} \text{(ii) Light travels at } 1.86 \times 10^5 \text{ miles per second} \\ = 186000 \text{ miles per second} \end{aligned}$$

8680000000000000000000000000 kg

(iii) 0.10524

Solution:

(iv) The distance between the sun and the planet Saturn 1.4335×10^{12} miles, which is already in scientific notation.

$\therefore$ No changes occurs.

$$123.456 = 1.23456 \times 10^2$$

3. Evaluate:

i) $\left(\frac{1}{2}\right)^3$ ii) $\left(\frac{1}{2}\right)^{-5}$ iii) $\left(\frac{-5}{6}\right)^{-3}$
 iv) $(2^{-5} \times 2^7) \div 2^{-2}$ v) $(2^{-1} \times 3^{-1}) \div 6^{-2}$

Solutions:

i) $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

ii) $\left(\frac{1}{2}\right)^{-5} = 25$
 $= 2 \times 2 \times 2 \times 2 \times 2$
 $= 32$

iii) $\left(\frac{-5}{6}\right)^{-3} = \left(\frac{-6}{5}\right)^3$
 $= \left(\frac{-6}{5}\right) \times \left(\frac{-6}{5}\right) \times \left(\frac{-6}{5}\right)$
 $= \frac{-216}{125}$

iv) $(2^{-5} \times 2^7) \div 2^{-2}$
 $= 2^{-5+7} \div 2^{-2}$
 $= 2^2 \div 2^{-2}$
 $= 2^2 \times 2^2 = 2^4$
 $= 16$

v) $(2^{-1} \times 3^{-1}) \div 6^{-2}$
 $= \left(\frac{1}{2} \times \frac{1}{3}\right) \div \left(\frac{1}{6}\right)^2$
 $= \frac{1}{6} \times \left(\frac{6}{1}\right)^2 = \frac{1}{6} \times 6 \times 6$
 $= 6$

4. Evaluate: i) $\left(\frac{2}{5}\right)^4 \times \left(\frac{5}{2}\right)^{-2}$

ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}$ iii) $2^7 \times \left(\frac{1}{2}\right)^{-3}$

Solution:

i) $\left(\frac{2}{5}\right)^4 \times \left(\frac{5}{2}\right)^{-2}$
 $= \left(\frac{2}{5}\right)^4 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^6 = \frac{64}{15625}$

ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3} = \left(\frac{4}{5}\right)^{-2} \times \left(\frac{4}{5}\right)^3$
 $= \left(\frac{4}{5}\right)^{(-2)+(3)}$
 $= \left(\frac{4}{5}\right)$

iii) $2^7 \times \left(\frac{1}{2}\right)^{-3} = 2^7 \times \left(\frac{2}{1}\right)^3 = 2^{10} = 1024$

5. Evaluate:

i) $(5^0 + 6^{-1}) \times 3^2$ ii) $(2^{-1} \times 3^{-1}) \div 6^{-1}$
 iii) $(3^{-1} + 4^{-2} + 5^{-3})^0$

Solutions:

i) $(5^0 + 6^{-1}) \times 3^2 = \left(1 + \frac{1}{6}\right) \times 3^2$
 $= \left(\frac{6+1}{6}\right) \times 3 \times 3$
 $= \frac{7}{6} \times 3 \times 3 = \frac{21}{2}$

ii) $(2^{-1} \times 3^{-1}) \div 6^{-1} = (2^{-1} + 3^{-1}) \div 6^{-1}$
 $= \left(\frac{1}{2} + \frac{1}{3}\right) \div \frac{1}{6}$
 $= \frac{3+2}{6} \times \frac{6}{1}$
 $= 5$

iii) $(3^{-1} + 4^{-2} + 5^{-3})^0 = 1$
 $\because a \neq 0, a^0 = 1$

6. Simplify: (i) $(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2$

(ii) $\frac{9^2 \times 7^3 \times 2^5}{84^3}$ (iii) $\frac{2^8 \times 2187}{3^5 \times 32}$

Solution:

(i) $(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2$
 $= 3^6 \times 2^{-2} \times 3^{-10} \times (2 \times 3^2)^2$
 $= 3^6 \times 2^{-2} \times 3^{-10} \times 2^2 \times 3^4$
 $= 3^{6+(-10)+4} \times 2^{(-2)+2}$
 $= 3^0 \times 2^0$
 $= 1 \times 1$
 $= 1$

$$\begin{aligned}
 \text{(ii)} \quad \frac{9^2 \times 7^3 \times 2^5}{(84)^3} &= \frac{(3^2)^2 \times 7^3 \times 2^5}{(7 \times 2^2 \times 3)^3} \\
 &= \frac{3^4 \times 7^3 \times 2^5}{7^3 \times 2^6 \times 3^3} \\
 &= 3^{4-3} \times 7^{3-3} \times 2^{5-6} \\
 &= 3^1 \times 7^0 \times 2^{-1} \\
 &= 3 \times 1 \times \frac{1}{2} = \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{2^8 \times 2187}{3^5 \times 32} &= \frac{2^8 \times 3^7}{3^5 \times 2^5} \\
 &= 2^8 \times 3^7 \times 3^{-5} \times 2^{-5} \\
 &= 2^{8-5} \times 3^{7-5} \\
 &= 2^3 \times 3^2 \\
 &= 8 \times 9 \\
 &= 72
 \end{aligned}$$

7. Solve for x :

$$\text{(i)} \quad \frac{2^{2x-1}}{2^{x+2}} = 4 \quad \text{(ii)} \quad \frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} = 5^{-5}$$

Solution:

$$\begin{aligned}
 \text{(i)} \quad \frac{2^{2x-1}}{2^{x+2}} &= 4 \\
 2^{2x-1} \times 2^{-(x+2)} &= 2^2 \\
 2^{(2x-1) + (-x-2)} &= 2^2 \\
 2^{2x-1-x-2} &= 2^2 \\
 2^{x-3} &= 2^2 \\
 x-3 &= 2 \\
 x &= 5
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} &= 5^{-5} \\
 5^{5-4+x-12} &= 5^{-5} \\
 5^{1+x-12} &= 5^{-5} \\
 1+x-12 &= -5 \\
 x-11 &= -5 \\
 x &= 11-5 \\
 x &= 6
 \end{aligned}$$

8. Expand using exponents:

(i) 6054.321

(ii) 897.14

Solution:

(i) 6054.321

$$\begin{aligned}
 &= 6 \times 1000 + 5 \times 10 + 4 \times 1 + 3 \times \frac{1}{10} + 2 \times \frac{1}{100} + 1 \times \frac{1}{1000} \\
 &= 6 \times 10^3 + 5 \times 10^1 + 4 \times 10^0 + 3 \times 10^{-1} + 2 \times 10^{-2} + 1 \times 10^{-3}
 \end{aligned}$$

(ii) 897.14

$$\begin{aligned}
 &= 8 \times 100 + 9 \times 10 + 7 \times 1 + 1 \times \frac{1}{10} + 4 \times \frac{1}{100} \\
 &= 8 \times 10^2 + 9 \times 10^1 + 7 \times 10^0 + 1 \times 10^{-1} + 4 \times 10^{-2}
 \end{aligned}$$

9. Find the number in standard form for the following expansions:

$$\text{(i)} \quad 8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$$

$$\text{(ii)} \quad 5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3}$$

$$\text{(iii)} \quad \text{The radius of a hydrogen atom is } 2.5 \times 10^{-11} \text{ m}$$

Solution:

$$\begin{aligned}
 \text{(i)} \quad &8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4} \\
 &= 8 \times 10000 + 7 \times 1000 + 6 \times 100 + 5 \times 10 + 2 + 4 \times \frac{1}{100} + 7 \times \frac{1}{10000} \\
 &= 80000 + 7000 + 600 + 50 + 2 + \frac{4}{100} + \frac{7}{10000} \\
 &= 87652.0407
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad &5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3} \\
 &= 5 \times 1000 + 5 \times 10 + 5 \times \frac{1}{10} + 5 \times \frac{1}{1000} \\
 &= 5000 + 50 + \frac{5}{10} + \frac{5}{1000} \\
 &= 5050.505
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad &\text{The standard form of } 2.5 \times 10^{-11} \\
 &= 2.5 \times \frac{1}{10^{11}} \\
 &= 0.000000000025
 \end{aligned}$$

10. Write the following numbers in scientific notation:

- (i) 467800000000
- (ii) 0.000001972
- (iii) 1642.398
- (iv) Earth's volume is about 1,083,000,000,000 cubic kilometres
- (v) If you fill a bucket with dirt, the portion of the whole Earth that is in the bucket will be 0.00000000000000000000000016 kg

Solution:

- (i) $467800000000 = 4.678 \times 10^{11}$
- (ii) $0.000001972 = 1.972 \times 10^{-6}$
- (iii) $1642.398 = 1.642398 \times 10^3$
- (iv) $1,083,000,000,000 \text{ cubic kilometers} = 1.083 \times 10^{12} \text{ cubic kilometers}$
- (v) $0.00000000000000000000000016 \text{ kg.} = 1.6 \times 10^{-24} \text{ kg.}$

11. By what number should $(-4)^{-1}$ be multiplied so that the product becomes 10^{-1} ?

- a) $\frac{2}{3}$
- b) $\frac{-2}{5}$
- c) $\frac{5}{2}$
- d) $\frac{-5}{2}$

Ans: b) $\frac{-2}{5}$

Solution:

Let the number be x

$$x(-4)^{-1} = 10^{-1}$$

$$x \left(\frac{-1}{4} \right) = \frac{1}{10}$$

$$\begin{aligned} x &= \frac{1}{10} \times \left(\frac{-4}{1} \right) \\ &= \frac{-2}{5} \end{aligned}$$

12. $(-2)^{-3} \times (-2)^{-2} =$

- a) $\frac{-1}{32}$
- b) $\frac{1}{32}$
- c) 32
- d) -32

Ans: a) $\frac{-1}{32}$

Solution:

$$\begin{aligned} (-2)^{-3} \times (-2)^{-2} &= \left(\frac{-1}{2} \right)^3 \times \left(\frac{-1}{2} \right)^2 \\ &= \frac{-1}{8} \times \frac{1}{4} \\ x &= \frac{-1}{32} \end{aligned}$$

13. Which is not correct?

- a) $\left(\frac{-1}{4} \right)^2 = 4^{-2}$
- b) $\left(\frac{-1}{4} \right)^2 = \left(\frac{1}{2} \right)^4$
- c) $\left(\frac{-1}{4} \right)^2 = 16^{-1}$
- d) $-\left(\frac{1}{4} \right)^2 = 16^{-1}$

Ans: d) $-\left(\frac{1}{4} \right)^2 = 16^{-1}$

Solution:

$$\begin{aligned} \text{a) } \left(\frac{-1}{4} \right)^2 &= 4^{-2} \\ &= \left(\frac{1}{4} \right)^2 = \left(\frac{1}{4} \right)^2 \left(\frac{1}{4} \right)^2 \end{aligned}$$

This is true.

$$\begin{aligned} \text{b) } \left(\frac{-1}{4} \right)^2 &= \left(\frac{1}{2} \right)^4 \\ &= \left(\left(\frac{1}{2} \right)^2 \right)^2 \end{aligned}$$

$$\left(\frac{-1}{4} \right)^2 = \left(\frac{1}{4} \right)^2$$

$$\left(\frac{1}{4} \right)^2 = \left(\frac{1}{4} \right)^2$$

This is true.

$$\begin{aligned} \text{c) } \left(\frac{-1}{4} \right)^2 &= 16^{-1} \\ &= \left(\frac{1}{16} \right) \end{aligned}$$

$$\left(\frac{-1}{4}\right)^2 = \left(\frac{1}{4}\right)^2$$

$$\left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2$$

This is true

d) $-\left(\frac{1}{4}\right)^2 = 16^{-1}$

$$-\left(\frac{1}{4}\right)^2 = \left(\frac{1}{16}\right)$$

$$-\left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 \quad \text{This is not true.}$$

14. If $\frac{10^x}{10^{-3}} = 10^9$, then x is _____.

- a) 4 b) 5 c) 6 d) 7

Solution:

Ans: (c) 6

$$\frac{10^x}{10^{-3}} = 10^9$$

$$10^{x-(-3)} = 10^9$$

$$10^{x+3} = 10^9$$

$$x+3 = 9$$

$$x = 9 - 3 = 6$$

15. 0.0000000002020 in scientific form is

- a) 2.02×10^9 b) 2.02×10^{-9}
c) 2.02×10^{-8} d) 2.02×10^{-10}

Ans: d) 2.02×10^{-10}

EXERCISE 1.7

MISCELLANEOUS PRACTICE PROBLEMS

1. If $\frac{3}{4}$ of a box of apples weighs 3 kg and 225 gm, how much does a full box of apples weigh?

Solution:

$$\frac{3}{4} \text{ of a box of apples weighs} = 3 \text{ kg } 225 \text{ gm}$$

$$= 3.225 \text{ kg}$$

$$\begin{aligned} \text{Full box of apples weigh} &= 3.225 \times \frac{4}{3} \\ &= \frac{12.9}{3} \\ &= 4.3 \text{ kg} \\ &= 4 \text{ kg } 300 \text{ gm} \end{aligned}$$

2. Mangalam buys a water jug of capacity $3\frac{4}{5}$ litre. If she buys another jug which is

$2\frac{2}{3}$ times as large as the smaller jug, how many litre can the larger one hold?

Solution:

$$\text{Capacity of the smaller jug} = 3\frac{4}{5} \text{ litres}$$

$$\text{Capacity of the larger jug} = 2\frac{2}{3} \text{ times of smaller jug}$$

$$= 2\frac{2}{3} \times 3\frac{4}{5}$$

$$= \frac{8}{3} \times \frac{19}{5}$$

$$= \frac{152}{15} = 10\frac{2}{15}$$

The capacity of the larger jug is $10\frac{2}{15}$ litre.

3. Ravi multiplied $\frac{25}{8}$ and $\frac{16}{15}$ and he says that the simplest form of this product is $\frac{10}{3}$ and Chandru says the answer in the simplest form is $3\frac{1}{3}$. Who is correct? (or)

Are they both correct? Explain.

Solution:

$$\begin{aligned}\text{Ravi's calculation} &= \frac{25}{8} \times \frac{16}{15} \\ &= \frac{400}{120} = \frac{10}{3} = 3\frac{1}{3}\end{aligned}$$

$$\text{Chandru's answer is } 3\frac{1}{3}$$

Both their answers are correct.

4. Find the length of a room whose area is $\frac{153}{10}$ sq.m and whose breadth is $2\frac{11}{20}$ m.

Solution:

$$\text{Area of the room} = \frac{153}{10} \text{ sq.m}$$

$$\text{Breadth of the room} = 2\frac{11}{20} \text{ m} = \frac{51}{20} \text{ m}$$

$$\text{Length} \times \text{Breadth} = \text{Area}$$

$$\text{Length} \times \frac{51}{20} \text{ m} = \frac{153}{10} \text{ sq.m}$$

$$\therefore \text{Length} = \frac{153}{10} \div \frac{51}{20}$$

$$= \frac{153}{10} \times \frac{20}{51}$$

$$= \frac{3}{1} \times \frac{2}{1} = 6 \text{ m.}$$

$\therefore$ The length of the room = 6 m

5. There is a large square portrait of a leader that covers an area of 4489 cm^2 . If each side has a 2 cm liner, what would be its area?

Solution:

$$\begin{aligned}\text{Area of the large square portrait} &= 4489 \text{ m}^2 \\ \text{i.e. } a \times a &= 4489\end{aligned}$$

$$\begin{array}{r} 67 \\ 6 \overline{) 4489} \\ \underline{36} \\ 889 \\ 127 \overline{) 889} \\ \underline{889} \\ 0 \end{array} \quad a = 67$$

$$\therefore \text{Length of the portrait} = 67 \text{ cm}$$

It has a 2 cm liner on each side.

$$\therefore \text{Total length} = 67 + 2 + 2 = 71 \text{ cm}$$

$$\begin{aligned}\text{The area of the square portrait} &= 71^2 \\ \text{including the liner} &= 71 \times 71\end{aligned}$$

$$= 5041 \text{ cm}^2$$

$$\therefore \text{The area of the liner} = 5041 - 4489 = 552 \text{ cm}^2$$

6. A greeting card has an area 90 cm^2 . Between what two whole numbers is the length of its side?

Solution:

$$\text{Area of the card} = 90 \text{ cm}^2$$

$$\therefore \text{Length of the card} = \sqrt{90} \text{ cm}^2$$

$$= 9.4$$

$$= 9 < 9.4 < 10$$

$\therefore$ The length of its side lies between 9 cm and 10 cm.

$$\begin{array}{r} 9.4 \\ 9 \overline{) 90.00} \\ \underline{81} \\ 900 \\ 184 \overline{) 900} \\ \underline{736} \\ 164 \end{array}$$

7. 225 square shaped mosaic tiles, each of area 1 square decimetre exactly cover a square shaped verandah. How long is each side of the square shaped verandah?

Solution:

Each square mosaic

$$\text{tiles area} = 1 \text{ square decimetre}$$

$\therefore$ 225 square mosaic

$$\text{tiles area} = 225 \text{ sq.decimetre}$$

$$\begin{aligned}\text{Area of the verandah} &= \text{Area of the 225} \\ &\text{mosaic tiles.}\end{aligned}$$

$$\begin{aligned}\text{Length of the verandah} &= \sqrt{225} \text{ decimetre} \\ &= 15 \text{ decimetre}\end{aligned}$$

8. If $\sqrt[3]{1906624} \times \sqrt{x} = 3100$, find x .

Solution:

$$\sqrt[3]{1906624} \times \sqrt{x} = 3100$$

4	1906624
4	476656
4	119164
31	29791
31	961
	31

$$\sqrt[3]{4 \times 4 \times 4 \times 31 \times 31 \times 31} \times \sqrt{x} = 3100$$

$$\sqrt[3]{4^3 \times 31^3} \times \sqrt{x} = 3100$$

$$\sqrt[3]{(4 \times 31)^3} \times \sqrt{x} = 3100$$

$$4 \times 31 \times \sqrt{x} = 3100$$

$$\sqrt{x} = \frac{3100}{31 \times 4}$$

$$\sqrt{x} = 25$$

Squaring on both sides,

$$x = 25^2$$

$$x = 25 \times 25$$

$$x = 625$$

9. If $2^{m-1} + 2^{m+1} = 640$, then find m .

Solution:

$$2^{m-1} + 2^{m+1} = 640$$

$$2^m \times 2^{-1} + 2^m \times 2^1 = 640$$

$$2^m (2^{-1} + 2) = 640$$

$$2^m \left(\frac{1}{2} + 2 \right) = 640$$

$$2^m \left(\frac{5}{2} \right) = 640$$

$$2^m = \frac{640 \times 2}{5}$$

$$2^m = 256$$

$$2^m = 2^8$$

$$m = 8$$

10. Give the answer in scientific notation:
A human heart beats at an average of 80 beats per minute. How many times does it beat in i) an hour? ii) a day?
iii) a year? iv) 100 years?

Solution:

Given that 1 minute = 80 beats

(i) an hour

$$\begin{aligned} 60 \text{ minutes} &= 80 \times 60 \text{ beats} \\ &= 4800 \text{ beats} \\ &= 4.8 \times 10^3 \text{ beats} \end{aligned}$$

(ii) a day

$$\begin{aligned} 60 \text{ minutes} &= 4800 \text{ beats} \\ \text{i.e) 1 hour} &= 4800 \text{ beats} \\ \therefore \text{ a day} &= 24 \times 4800 \text{ beats} \\ &= 115200 \text{ beats} \\ &= 1.152 \times 10^5 \text{ beats} \end{aligned}$$

(iii) a year

$$\begin{aligned} \text{a day} &= 115200 \text{ beats} \\ \text{a year} &= 365 \text{ days} \\ \therefore \text{ a year} &= 365 \times 115200 \text{ beats} \\ &= 42048000 \text{ beats} \\ &= 4.2048 \times 10^7 \text{ beats} \end{aligned}$$

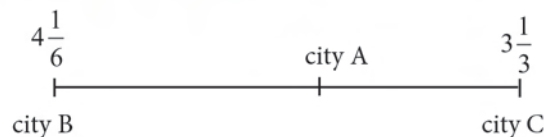
(iv) 100 years

$$\begin{aligned} \text{a year} &= 42048000 \text{ beats} \\ 100 \text{ years} &= 42048000 \times 100 \text{ beats} \\ &= 4204800000 \text{ beats} \\ &= 4.2048 \times 10^9 \text{ beats} \end{aligned}$$

CHALLENGING PROBLEMS

11. In a map, if 1 inch refers to 120 km, then find the distance between two cities B and C which are $4\frac{1}{6}$ inches and $3\frac{1}{3}$ inches from the city A which lies between the cities B and C.

Solution:



From the figure, the total inches between the two cities B and C

$$= 4\frac{1}{6} + 3\frac{1}{3} = \frac{25}{6} + \frac{10}{3}$$

$$= \frac{25+20}{6} = \frac{45}{6} \text{ inches}$$

1 inch refers to 120 km

$$\therefore \frac{45}{6} \text{ inches} = \frac{45}{6} \times 120$$

$$= 45 \times 20$$

$$= 900 \text{ km}$$

The distance between the cities B and C is 900 km.

12. Give an example and verify each of the following statements.

- The collection of all non-zero rational numbers is closed under division.
- Subtraction is not commutative for rational numbers.
- Division is not associative for rational numbers.
- Distributive property of multiplication over subtraction is true for rational numbers. That is, $a(b-c) = ab - ac$.
- The mean of two rational numbers is rational and lies between them.

Solution:

i) Let $a = \frac{3}{5}, b = \frac{-2}{7}$

$$a \div b = \frac{3}{5} \div \frac{-2}{7} = \frac{3}{5} \times \frac{7}{-2} = \frac{-21}{10}$$

$\frac{-21}{10}$ is also a rational number.

ie) The collection of all non-zero rational numbers is closed under division.

ii) Let $a = \frac{5}{7}; b = \frac{4}{5}$

$$a - b = \frac{5}{7} - \frac{4}{5} = \frac{25-28}{35}$$

$$= \frac{-3}{35} \quad \dots\dots\dots (1)$$

$$b - a = \frac{4}{5} - \frac{5}{7} = \frac{28-25}{35}$$

$$= \frac{3}{35} \quad \dots\dots\dots (2)$$

From (1) and (2), $a - b \neq b - a$

ie) Subtraction is not commutative for rational numbers.

iii) Let $a = \frac{1}{2}; b = \frac{2}{3}; c = \frac{3}{4}$

$$a \div b = \frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$(a \div b) \div c = \frac{3}{4} \div \frac{3}{4} = \frac{3}{4} \times \frac{4}{3} = 1 \quad \dots\dots\dots (1)$$

$$b \div c = \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

$$a \div (b \div c) = \frac{1}{2} \div \frac{8}{9} = \frac{1}{2} \times \frac{9}{8}$$

$$= \frac{9}{16} \quad \dots\dots\dots (2)$$

from (1) and (2);

$$(a \div b) \div c \neq a \div (b \div c)$$

Division is not associative for rational numbers.

iv) Let $a = \frac{5}{3}, b = \frac{2}{3}, c = \frac{1}{7}$

$$a(b-c) = \frac{5}{3} \left(\frac{2}{3} - \frac{1}{7} \right) = \frac{5}{3} \left(\frac{14-3}{21} \right)$$

$$= \frac{5}{3} \times \frac{11}{21} = \frac{55}{63} \quad \dots\dots\dots (1)$$

$$ab - ac = \frac{5}{3} \times \frac{2}{3} - \frac{5}{3} \times \frac{1}{7}$$

$$= \frac{10}{9} - \frac{5}{21} = \frac{70-15}{63}$$

$$= \frac{55}{63} \quad \dots\dots\dots (2)$$

From (1) and (2), Distributive property of multiplication over subtraction is true for rational numbers. That is, $a(b - c) = ab - ac$

$$\text{v) Let } a = \frac{3}{5}, b = \frac{4}{5}$$

$$\text{Mean} = \frac{1}{2}(a + b)$$

$$= \frac{1}{2}\left(\frac{3}{5} + \frac{4}{5}\right) = \frac{1}{2}\left(\frac{3+4}{5}\right)$$

$$= \frac{1}{2} \times \frac{7}{5}$$

$$\text{Mean} = \frac{7}{10}$$

$$a = \frac{3}{5} = \frac{42}{70}$$

$$b = \frac{4}{5} = \frac{56}{70},$$

$$\text{Mean} = \frac{7}{10} = \frac{49}{70}$$

$$\text{ie) } \frac{42}{70} < \frac{49}{70} < \frac{56}{70}$$

$$\text{ie) } \frac{3}{5} < \frac{7}{10} < \frac{4}{5}$$

That is, the mean of two rational numbers lies between them.

13. If $\frac{1}{4}$ of a ragi adai weighs 120 grams, what will be the weight of $\frac{2}{3}$ of the same ragi adai?

Solution:

$$\frac{1}{4} \text{ of a ragi adai weighs 120 grams.}$$

$$\therefore \text{The weight of the full ragi adai} \\ = 4 \times 120 = 480 \text{ gm}$$

$$\therefore \text{The weight of } \frac{2}{3} \text{ of the ragi adai}$$

$$= \frac{2}{3} \times 480 = 320 \text{ gm}$$

14. If $p + 2q = 18$ and $pq = 40$ find $\frac{2}{p} + \frac{1}{q}$.

Solution:

$$p + 2q = 18 \text{ and } pq = 40$$

$$\frac{2}{p} + \frac{1}{q} = \frac{2q + p}{pq} = \frac{18}{40} = \frac{9}{20}$$

15. Find 'x' if $5\frac{x}{5} \times 3\frac{3}{4} = 21$

Solution:

$$5\frac{x}{5} \times 3\frac{3}{4} = 21$$

$$\frac{25+x}{5} \times \frac{15}{4} = 21$$

$$(25+x) \frac{3}{4} = 21$$

$$25+x = 21 \times \frac{4}{3}$$

$$25+x = 28$$

$$x = 28 - 25$$

$$x = 3$$

16. By how much does $\left(\frac{10}{11}\right)$ exceed $\left(\frac{1}{10}\right)$?

Solution:

$$\frac{1}{\left(\frac{10}{11}\right)} - \left(\frac{1}{10}\right) = \frac{11}{10} - \frac{1}{10} \times \frac{1}{11}$$

$$= \frac{11}{10} - \frac{1}{110} = \frac{121-1}{110}$$

$$= \frac{120}{110} = \frac{12}{11}$$

17. A group of 1536 cadets wanted to have a parade forming a square design. Is it possible? If it is not possible how many more cadets would be required?

Solution:

$$\text{Number of cadets} = 1536$$

$$= 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

Since '3' and '2' do not have pair.

∴ We cannot form a square design of 1536 cadets.

The next perfect square number of 1536 is 1600.

∴ The required number of cadets
= 1600 - 1536
= 64

3	1536
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

18. Evaluate: $\sqrt{286225}$ and use it to compute $\sqrt{2862.25} + \sqrt{28.6225}$

Solution:

$$\sqrt{286225} = 535$$

$$\sqrt{2862.25} = 53.5$$

$$\sqrt{28.6225} = 5.35$$

$$\sqrt{2862.25} + \sqrt{28.6225} = 53.5 + 5.35 \\ = 58.85$$

19. Simplify: $(3.769 \times 10^5) + (4.21 \times 10^5)$

Solution:

$$(3.769 \times 10^5) + (4.21 \times 10^5) = 376900 + 421000 \\ = 797900 \\ = 7.979 \times 10^5$$

20. Order the following from the least to the greatest: 16^{25} , 8^{100} , 3^{500} , 4^{400} , 2^{600}

Solution:

$$16^{25} = (16)^{\frac{25 \times 4}{4}} = (\sqrt[4]{16})^{100} = (2)^{100}$$

$$8^{100} = (8)^{100}$$

$$3^{500} = 3^{5 \times 100} = (3^5)^{100} = (243)^{100}$$

$$4^{400} = 4^{4 \times 100} = (256)^{100}$$

$$2^{600} = 2^{6 \times 100} = (64)^{100}$$

As the powers of all numbers are 100, we compare the base of the numbers.

∴ The ascending order is 16^{25} , 8^{100} , 2^{600} , 3^{500} , 4^{400} .



CREATIVE QUESTIONS

I. CHOOSE THE BEST ANSWER:

1. When 0.15 is written in simplest fractional form, the sum of the numerator and the denominator is

- a) 115 b) 46
c) 23 d) 22 **Ans: c) 23**

Solution:

$$0.15 = \frac{15}{100} = \frac{3}{20}$$

∴ Sum of the numerator and denominator
= 3 + 20 = 23

2. The value of $\frac{55}{100000}$ in decimal form is

- a) 0.00055 b) 0.0055
c) 0.055 d) 0.5500

Ans: a) 0.00055

Solution:

$$\frac{55}{100000} = 0.00055$$

3. How many times is 0.1 greater than 0.01?

- a) 1 time b) 10 times
c) 100 times d) 1000 times

Ans: b) 10 times

Solution:

$$0.1 = \frac{1}{10} = \frac{10}{100} = \left(\frac{1}{100}\right)10$$

$$0.1 = (0.01) \times 10$$

4. Which of the following fractions is less than $\frac{1}{5}$?

- a) $\frac{8}{35}$ b) $\frac{8}{37}$
 c) $\frac{8}{39}$ d) $\frac{2}{11}$ **Ans: d)** $\frac{2}{11}$

Solution:

$$\frac{1}{5} = 0.2000$$

$$\frac{8}{35} = 0.2285$$

$$\frac{8}{37} = 0.2162$$

$$\frac{8}{39} = 0.2051$$

$$\frac{2}{11} = 0.1818$$

$$\frac{2}{11} \text{ is less than } \frac{1}{5}$$

5. If $\frac{1}{1.25} = 0.8$ then the value of $\frac{1}{0.0125}$

- a) 8 b) 0.008
 c) 0.8 d) 80 **Ans: d)** 80

Solution:

$$\frac{1}{0.0125} = \frac{1}{1.25} \times 100 = 0.8 \times 100 = 80$$

6. The correct ascending order of given fraction is

- a) $\frac{5}{7}, \frac{9}{11}, \frac{7}{8}$ b) $\frac{5}{7}, \frac{7}{8}, \frac{9}{11}$
 c) $\frac{7}{8}, \frac{5}{7}, \frac{9}{11}$ d) $\frac{9}{11}, \frac{7}{8}, \frac{5}{7}$

Ans: a) $\frac{5}{7}, \frac{9}{11}, \frac{7}{8}$

Solution:

$$\frac{5}{7} = 0.71, \frac{9}{11} = 0.81, \frac{7}{8} = 0.87$$

$\therefore$ The ascending order $\frac{5}{7}, \frac{9}{11}, \frac{7}{8}$

7. What is the value of x in the following

$$x + \frac{6}{8} + 3\frac{1}{4} = 23\frac{13}{24}?$$

- a) $11\frac{1}{8}$ b) $12\frac{1}{8}$
 c) $16\frac{1}{4}$ d) None of these
Ans: d) None of these

Solution:

$$x + \frac{6}{8} + 3\frac{1}{4} = 23\frac{13}{24}$$

$$x + \frac{6}{8} + \frac{13}{4} = \frac{565}{24}$$

$$x + \frac{6+26}{8} = \frac{565}{24}$$

$$x + \frac{32}{8} = \frac{565}{24}$$

$$x + 4 = \frac{565}{24}$$

$$x = \frac{565}{24} - 4$$

$$= \frac{565-96}{24} = \frac{469}{24}$$

$$= 19\frac{13}{24}$$

8. $3\left[\left(\frac{1}{2} + \frac{3}{4}\right) \div \left(\frac{1}{2} - \frac{1}{4}\right)\right] =$

- a) 5 b) 15 c) 0 d) 25
Ans: b) 15

Solution:

$$3\left[\left(\frac{1}{2} + \frac{3}{4}\right) \div \left(\frac{1}{2} - \frac{1}{4}\right)\right]$$

$$= 3\left[\left(\frac{2+3}{4}\right) \div \left(\frac{2-1}{4}\right)\right]$$

$$= 3\left(\frac{5}{4} \div \frac{1}{4}\right) = 3\left(\frac{5}{4} \times \frac{4}{1}\right) = 15$$

9. $3\frac{1}{2} - 2\frac{3}{5} - 7\frac{3}{4} + 5\frac{1}{4} =$

a) $\frac{-33}{20}$

b) $\frac{-8}{5}$

c) $\frac{34}{-20}$

d) $\frac{+8}{5}$ **Ans: b)** $\frac{-8}{5}$

Solution:

$$\begin{aligned} 3\frac{1}{2} - 2\frac{3}{5} - 7\frac{3}{4} + 5\frac{1}{4} \\ = \frac{7}{2} - \frac{13}{5} - \frac{31}{4} + \frac{21}{4} \\ = \frac{70 - 52 - 155 + 105}{20} \\ = \frac{175 - 207}{20} = \frac{-32}{20} = \frac{-8}{5} \end{aligned}$$

II. FILL IN THE BLANKS

1. The simplest form of $\frac{576}{360}$ is _____. **Ans:** $\frac{72}{45}$

Solution:

The simplest form of $\frac{576}{360} = \frac{72}{45}$

2. The mixed form of $\frac{-63}{8}$ is _____. **Ans:** $-7\frac{7}{8}$

Solution:

The mixed form of $\frac{-63}{8} = -7\frac{7}{8}$

3. $\left(-5\frac{2}{3}\right) \times \left(-2\frac{3}{4}\right) =$ _____. **Ans:** $\frac{187}{12}$

Solution:

$$\begin{aligned} \left(-5\frac{2}{3}\right) \times \left(-2\frac{3}{4}\right) \\ = \left(\frac{-17}{3}\right) \times \left(\frac{-11}{4}\right) = \frac{187}{12} \end{aligned}$$

4. $\left(3\frac{1}{7}\right) \div \left(\frac{11}{315}\right) =$ _____. **Ans:** 90

Solution:

$$\begin{aligned} \left(3\frac{1}{7}\right) \div \left(\frac{11}{315}\right) &= \left(\frac{22}{7}\right) \times \frac{315}{11} \\ &= 2 \times 45 = 90 \end{aligned}$$

5. If $-3.5 = \frac{x}{10}$ then the value of x is _____. **Ans:** -35

Solution:

$$\begin{aligned} -3.5 &= \frac{x}{10} \\ \therefore x &= -3.5 \times 10 = -35 \end{aligned}$$

6. If $\frac{5}{7} = \frac{a}{42}$ then the value of a is _____. **Ans:** 30

Solution:

$$\begin{aligned} \frac{5}{7} &= \frac{a}{42} \\ 7a &= 5 \times 42 \\ a &= \frac{5 \times 42}{7} \\ a &= 30 \end{aligned}$$

7. $-8\frac{1}{4}$ lies between the integers _____ and _____. **Ans:** -8 and -9

8. The smallest rational number is _____. **Ans:** not defined

9. The multiplicative inverse of $-7\frac{2}{5}$ is _____. **Ans:** $-\frac{5}{37}$

Solution:

Let x be the multiplicative inverse of $-7\frac{2}{5}$

$$\therefore -7\frac{2}{5} \times x = 1$$

$$-\frac{37}{5} \times x = 1$$

$$x = \frac{-5}{37}$$

10. If $\frac{7}{8} \times x = \frac{6}{-5} \times y$ then $\frac{x}{y}$ is **Ans:** $\frac{48}{-35}$

Solution:

$$\frac{7}{8} \times x = \frac{6}{-5} \times y$$

$$\therefore \frac{x}{y} = \frac{6}{-5} \times \frac{8}{7} = \frac{48}{-35}$$

III. 2 MARK QUESTIONS:

1. If cost of $\frac{2}{3}$ part of goods is ₹ 2200, then what is the cost of $\frac{3}{11}$ part of goods?

Solution:

The cost of $\frac{2}{3}$ part of goods is ₹ 2200.

$$\begin{aligned} \text{The cost of } \frac{3}{11} \text{ part of goods is} &= 2200 \times \frac{3}{2} \times \frac{3}{11} \\ &= ₹ 900 \end{aligned}$$

2. If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of the number then find the $\frac{5}{3}$ of the number.

Solution:

Let the number be x

By the given data

$$\frac{3}{4}x = 7 + \frac{1}{6}x$$

$$\frac{3x}{4} - \frac{x}{6} = 7$$

$$\frac{9x - 2x}{12} = 7$$

$$7x = 84$$

$$x = 12$$

$$\therefore \frac{5}{3} \text{ of the number is } \frac{5}{3} \times 12 = 20$$

3. Ravi consumes $\frac{2}{3}$ litre of milk everyday. How many litres of milk will he consume in 4 weeks?

Solution:

Ravi consumes $\frac{2}{3}$ litre of milk per day

Ravi consumes the milk in 4 weeks is

$$\begin{aligned} &= \frac{2}{3} \times 4 \times 7 \\ &= \frac{56}{3} = 18\frac{2}{3} \text{ litre.} \end{aligned}$$

4. From a tree $11\frac{1}{2}$ m long two pieces of length $3\frac{3}{4}$ m and $4\frac{5}{6}$ m are cut off. What is the length of the remaining tree?

Solution:

Length of the tree = $11\frac{1}{2}$ m

Length of two pieces = $3\frac{3}{4}$ m and $4\frac{5}{6}$ m

Total length of two pieces = $3\frac{3}{4} + 4\frac{5}{6}$

$$= \frac{15}{4} + \frac{29}{6} = \frac{45 + 58}{12} = \frac{103}{12}$$

$\therefore$ The length of the remaining tree

$$= 11\frac{1}{2} - \frac{103}{12} = \frac{23}{2} - \frac{103}{12}$$

$$= \frac{138 - 103}{12} = \frac{35}{12} \text{ m.}$$

5. A car is moving at an average speed of $\frac{350}{4}$ km/hr. How much distance will it cover in $\frac{5}{4}$ hours?

Solution:

$$\text{Speed of the car} = \frac{350}{4} \text{ km / hr}$$

Distance covered by the car in $\frac{5}{4}$ hours

$$\begin{aligned} &= \frac{350}{4} \times \frac{5}{4} \text{ km} \\ &= \frac{1750}{16} = 109\frac{3}{8} \text{ km} \end{aligned}$$

6. If $\sqrt{3^n} = 729$, then Find the value of n.

Solution:

$$\begin{aligned} \sqrt{3^n} &= \sqrt{3^6} \\ \text{Squaring on both sides} \\ 3^n &= 3^6 \\ \therefore n &= 6 \end{aligned}$$

3	729
3	243
3	81
3	27
3	9
	3

7. The area of a room is $\frac{261}{4} \text{ m}^2$. If its breadth is $\frac{87}{16} \text{ m}$ what is its length?

Solution:

$$\begin{aligned} \text{Area of the room} &= \frac{261}{4} \text{ m}^2 \\ \text{Breadth} &= \frac{87}{16} \text{ m} \\ \text{Length} &= \frac{261}{4} \div \frac{87}{16} \\ &= \frac{261}{4} \times \frac{16}{87} \\ &= \frac{3}{1} \times \frac{4}{1} = 12 \text{ m.} \end{aligned}$$

8. By what number should we multiply $\frac{-8}{13}$ so that the product may be 24?

Solution:

Let the number be x
By the given data

$$\begin{aligned} x \times \left(\frac{-8}{13}\right) &= 24 \\ x &= 24 \times \frac{-13}{8} \\ &= -39 \end{aligned}$$

IV. 4 MARK QUESTIONS:

1. Express 37^2 as the sum of two consecutive positive integers.

Solution:

$$\begin{aligned} 37^2 &= \frac{37^2-1}{2} + \frac{37^2+1}{2} \\ &= \frac{1369-1}{2} + \frac{1369+1}{2} \\ &= \frac{1368}{2} + \frac{1370}{2} \\ &= 684 + 685 \end{aligned}$$

2. Find the square root of 2601 by prime factors method.

Solution:

3	2601
3	867
17	289
17	17
	1

$$\begin{aligned} 2601 &= 3 \times 3 \times 17 \times 17 \\ &= 3^2 \times 17^2 \\ &= (3 \times 17)^2 \\ \therefore \sqrt{2601} &= 3 \times 17 \\ &= 51 \end{aligned}$$

3. Find the sum of the perfect squares between 120 and 300.

Solution:

The perfect squares between 120 and 300 are 121, 144, 169, 196, 225, 256, 289.

The sum of all perfect squares between 120 and 300

$$\begin{aligned} &= 121 + 144 + 169 + 196 + 225 + 256 + 289 \\ &= 1400 \end{aligned}$$

4. Two numbers are in the ratio of 3 : 4. Their L.C.M is 84. Find the numbers.

Solution:

Given that the ratio of numbers = 3 : 4

Let the numbers be $3x$ and $4x$.

$$\therefore 12x = 84 \text{ (given)}$$

$$x = \frac{84}{12}$$

$$x = 7$$

$\therefore$ The numbers are $3x = 3 \times 7 = 21$ and $4x = 4 \times 7 = 28$.

5. Evaluate: $\sqrt{0.0009}$

Solution:

$$\begin{aligned}\sqrt{0.0009} &= \sqrt{\frac{0.0009 \times 10000}{10000}} \\ &= \sqrt{\frac{9}{10000}} \\ &= \frac{\sqrt{9}}{\sqrt{10000}} = \frac{3}{100}\end{aligned}$$

$$\sqrt{0.0009} = 0.03$$

6. Find the value of x

$$\sqrt{86.49} + \sqrt{5 + x^2} = 12.3$$

Solution:

$$\sqrt{86.49} + \sqrt{5 + x^2} = 12.3$$

$$9.3 + \sqrt{5 + x^2} = 12.3$$

$$\sqrt{5 + x^2} = 12.3 - 9.3$$

$$\sqrt{5 + x^2} = 3$$

Squaring on both sides

$$5 + x^2 = 9$$

$$x^2 = 9 - 5$$

$$x^2 = 4$$

$$x = 2$$

$$\begin{array}{r} 9.3 \\ 9 \overline{) 86.49} \\ \underline{81} \\ 549 \\ 183 \overline{) 549} \\ \underline{549} \\ 0 \end{array}$$

7. If $\sqrt{15} = 3.88$. Find the value of $\sqrt{\frac{5}{3}}$

Solution:

$$\sqrt{15} = 3.88$$

$$\begin{aligned}\therefore \sqrt{\frac{5}{3}} &= \sqrt{\frac{5 \times 3}{3 \times 3}} \\ &= \frac{\sqrt{15}}{\sqrt{9}} \\ &= \frac{3.88}{3} \\ &= 1.29\end{aligned}$$

8. Find the cube root of 2744

Solution:

$$\begin{aligned}\sqrt[3]{2744} &= \sqrt[3]{7^3 \times 2^3} \\ &= \sqrt[3]{(7^3 \times 2^3)^3} \\ &= 7 \times 2 \\ \sqrt[3]{2744} &= 14\end{aligned}$$

2	2744
2	1372
2	686
7	343
7	49
	7

9. If $\sqrt{x} \div \sqrt{441} = 0.02$ then Find the value of x .

Solution:

$$\sqrt{x} \div \sqrt{441} = 0.02$$

$$\frac{\sqrt{x}}{\sqrt{441}} = 0.02$$

$$\frac{\sqrt{x}}{21} = 0.02$$

$$\begin{aligned}\sqrt{x} &= 0.02 \times 21 \\ &= 0.42\end{aligned}$$

Squaring on both sides

$$x = (0.42)^2$$

$$x = 0.1764$$

10. Simplify: $\frac{2^6 \times 3^3 \times 5^2}{40 \times 81 \times 125}$

Solution:

$$\begin{aligned}\frac{2^6 \times 3^3 \times 5^2}{40 \times 81 \times 125} &= \frac{2^6 \times 3^3 \times 5^2}{5 \times 2^3 \times 3^4 \times 5^3} \\ &= \frac{2^{6-3}}{3^{4-3} \times 5^{4-2}} \\ &= \frac{2^3}{3^1 \times 5^2} = \frac{8}{3 \times 25} = \frac{8}{75}\end{aligned}$$

11. Simplify: $\frac{800 \times 10^5}{1000}$

Solution:

$$\begin{aligned}\frac{800 \times 10^5}{1000} &= \frac{2 \times 10^2 \times 10^5}{10^3} \\ &= \frac{2 \times 10^7}{10^3} \\ &= 2 \times 10^4 \\ &= 2 \times 10000 \\ &= 20000\end{aligned}$$

V. 5 MARK QUESTIONS:

1. Arrange the following rational number in descending order $\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{5}{-4}, \frac{140}{28}$

Solution:

LCM of 8, 16, 12, 4, 28 = 336

$$\begin{aligned}\frac{7}{8} &= \frac{294}{336} \\ \frac{64}{16} &= \frac{1344}{336} \\ \frac{36}{-12} &= \frac{-1008}{336} \\ \frac{5}{-4} &= \frac{-420}{336} \\ \frac{140}{28} &= \frac{1680}{336}\end{aligned}$$

∴ The descending order

$$\frac{140}{28}, \frac{64}{16}, \frac{7}{8}, \frac{5}{-4}, \frac{36}{-12}$$

2. Solve: $2\frac{3}{5} + 4\frac{3}{4} - 5\frac{1}{4} = x + 3\frac{1}{2}$

Solution:

$$2\frac{3}{5} + 4\frac{3}{4} - 5\frac{1}{4} = x + 3\frac{1}{2}$$

$$\frac{13}{5} + \frac{19}{4} - \frac{21}{4} = x + \frac{7}{2}$$

$$\frac{13}{5} - \frac{2}{4} = x + \frac{7}{2}$$

$$\frac{52-10}{20} = x + \frac{7}{2}$$

$$\frac{42}{20} = x + \frac{7}{2}$$

$$\therefore x = \frac{42}{20} - \frac{7}{2}$$

$$= \frac{42-70}{20}$$

$$= \frac{-28}{20}$$

$$= \frac{-7}{5}$$

3. Simplify: $\left(2\frac{5}{6} \times 3\frac{1}{3}\right) \div \left(6\frac{2}{3} \times 5\frac{1}{4}\right)$

Solution:

$$\left(2\frac{5}{6} \times 3\frac{1}{3}\right) \div \left(6\frac{2}{3} \times 5\frac{1}{4}\right)$$

$$= \left(\frac{17}{6} \times \frac{10}{3}\right) \div \left(\frac{20}{3} \times \frac{21}{4}\right)$$

$$= \left(\frac{17}{3} \times \frac{5}{3}\right) \div \left(\frac{5}{1} \times \frac{7}{1}\right)$$

$$= \frac{85}{9} \div \frac{35}{1} = \frac{85}{9} \times \frac{1}{35} = \frac{17}{63}$$

4. Find the least number by which 3675 should be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

Solution:

$$\begin{array}{r|l}
 5 & 3675 \\
 \hline
 5 & 735 \\
 7 & 147 \\
 7 & 21 \\
 & 3
 \end{array}$$

$$3675 = 7 \times 7 \times 5 \times 5 \times 3$$

To get a perfect square 3 should be multiplied with 3675.

$$\text{i.e. } 3675 \times 3 = 11025$$

$$= 7 \times 7 \times 5 \times 5 \times 3 \times 3$$

$$= 7^2 \times 5^2 \times 3^2$$

$$\therefore \sqrt{11025} = 7 \times 5 \times 3 = 105$$

5. Consider any natural number $m > 1$. $2m$, $m^2 - 1$, $m^2 + 1$ will form a pythagorean triplet. Check this statement, if $m = 11$.

Solution:

Given that $2m$, $m^2 - 1$, $m^2 + 1$ as a pythagorean triplet and $m = 11$.

$$\therefore \text{ we have } 2 \times 11, 11^2 - 1, 11^2 + 1$$

$$= 22, 121 - 1, 121 + 1$$

$$= 22, 120, 122$$

Take 122 as the length of hypotenuse

$$120^2 + 22^2 = 14400 + 484$$

$$= 14884$$

$$= 122^2$$

Hence any natural number $m > 1$, $2m$, $m^2 - 1$, $m^2 + 1$ will form a Pythagorean triplet.

6. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

Solution:

The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively is H.C.F of $(1657 - 6)$ and $(2037 - 5)$

i.e. H.C.F of 1651 and 2032.

$$\begin{array}{r|l}
 1 & 2032 \\
 \hline
 1651 & \\
 \hline
 381 &
 \end{array}
 \quad
 \begin{array}{r|l}
 4 & 1651 \\
 \hline
 381 & 1524 \\
 \hline
 127 &
 \end{array}$$

$$\begin{array}{r|l}
 3 & 127 \\
 \hline
 381 & \\
 \hline
 0 &
 \end{array}$$

H.C.F of 1651 and 2032 = 3.

$\therefore$ The required number = 3.

7. Find the least number that must be subtracted to 1540 so as to get a perfect square. Also find the square root of the perfect square.

Solution:

$$\begin{array}{r|l}
 39.2 & \\
 3 & 1540.00 \\
 \hline
 & 9 \\
 69 & 640 \\
 & 621 \\
 782 & 1900 \\
 & 1564
 \end{array}$$

$$\sqrt{1540} = 39.2$$

The nearest square number of 1540 is 1521

The least number that must be subtracted from 1540, so as to get a perfect square

$$= 1540 - 1521$$

$$= 19$$

8. Evaluate: $\sqrt[3]{\frac{5832}{2744}}$

Solution:

2	2744	2	5832
2	1372	2	2916
2	686	2	1458
7	343	3	729
7	49	3	243
	7	3	81
		3	27
		3	9
			3

$$\begin{aligned}\sqrt[3]{\frac{5832}{2744}} &= \frac{\sqrt[3]{5832}}{\sqrt[3]{2744}} \\ &= \frac{\sqrt[3]{2^3 \times 3^3 \times 3^3}}{\sqrt[3]{2^3 \times 7^3}} \\ &= \frac{\sqrt[3]{(2 \times 3 \times 3)^3}}{\sqrt[3]{(2 \times 7)^3}} = \frac{2 \times 3 \times 3}{2 \times 7} \\ \sqrt[3]{\frac{5832}{2744}} &= \frac{18}{14}\end{aligned}$$

9. Evaluate: $\sqrt{175.2976}$

Solution:

	13.24
1	175.29 76
	1
23	75
	69
262	629
	524
2644	10576
	10576
	0

$$\therefore \sqrt{175.2976} = 13.24$$

10. Evaluate $\sqrt{0.9}$ upto 3 places of decimal

Solution:

$$\begin{array}{r} 0.9486 \\ 9 \overline{) 0.90 \, 00 \, 00 \, 00} \\ \underline{81} \\ 900 \\ \underline{736} \\ 16400 \\ \underline{15104} \\ 129600 \\ \underline{119196} \\ 0 \end{array}$$

$$\begin{aligned}\sqrt{0.9} &= 0.9486 \\ &= 0.949\end{aligned}$$

11. If $\sqrt{1 + \frac{x}{169}} = \frac{14}{13}$, Find the value of x .

Solution:

$$\begin{aligned}\sqrt{1 + \frac{x}{169}} &= \frac{14}{13} \\ \sqrt{\frac{169 + x}{169}} &= \frac{14}{13}\end{aligned}$$

$$\frac{\sqrt{169 + x}}{13} = \frac{14}{13}$$

$$\sqrt{169 + x} = 14$$

Squaring on both sides.

$$169 + x = 14^2$$

$$169 + x = 196$$

$$x = 196 - 169$$

$$= 27$$

UNIT TEST

Time : 1 hr

MARKS : 30

UNIT - 1 | NUMBERS

I. FILL IN THE BLANKS:

5 × 1 = 5

1. The simplest form of $\frac{160}{340}$ is _____.
2. If $\frac{3}{5} = \frac{x}{40}$ then x is _____.
3. The rational form of - 7.5 is _____.
4. The greatest rational number of $\frac{5}{-4}$ and $\frac{-11}{-7}$ is _____.
5. $\frac{-8}{5} - \frac{8}{-5} =$ _____

II. CHOOSE THE BEST ANSWER :

5 × 1 = 5

6. The numbers which is subtracted from $\frac{-6}{11}$ to get $\frac{8}{9}$ is
 - a) $\frac{34}{99}$
 - b) $\frac{-142}{99}$
 - c) $\frac{142}{99}$
 - d) $\frac{-34}{99}$
7. The sum of the digits of the denominator in the simplest form of $\frac{112}{528}$ is
 - a) 4
 - b) 5
 - c) 6
 - d) 7
8. $\frac{3}{4} \div \left(\frac{5}{8} + \frac{1}{2} \right) =$
 - a) $\frac{13}{10}$
 - b) $\frac{2}{3}$
 - c) $\frac{3}{2}$
 - d) $\frac{5}{8}$
9. Which of the following fraction lies between $\frac{2}{3}$ and $\frac{3}{5}$?
 - a) $\frac{2}{5}$
 - b) $\frac{1}{3}$
 - c) $\frac{1}{15}$
 - d) $\frac{31}{50}$

10. Out of the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{16}$ and $\frac{5}{8}$, which is the largest?

a) $\frac{5}{8}$

b) $\frac{3}{4}$

c) $\frac{5}{16}$

d) $\frac{1}{2}$

III. ANSWER THE FOLLOWING QUESTIONS:**5 × 2 = 10**

11. Find the value of $\frac{3}{5} + \frac{5}{8} + \frac{7}{10}$

12. Multiply $2\frac{3}{5}$ and $1\frac{4}{7}$

13. Verify the closure property for addition of the rational numbers $-\frac{5}{7}$ and $\frac{8}{9}$

14. Find the value of $\left(1 - \frac{1}{2}\right) \times \left(\frac{1}{2} - \frac{1}{4}\right) \div \left(\frac{3}{4} - \frac{1}{2}\right)$

15. If $pq = \frac{3}{5}$ and $q = \frac{2}{3}$ then find the value of $\frac{p}{2q} - \frac{3}{q}$.

IV. ANSWER THE FOLLOWING QUESTIONS:**2 × 5 = 10**

16. Verify the associative property for multiplication of the rational numbers $-\frac{3}{5}$, $\frac{4}{7}$, $\frac{2}{3}$

17. In a map, if 1 inch refers to 120 km then find the distance between two cities B and C which are $4\frac{1}{6}$ inches and $3\frac{1}{3}$ inches from the city A which lies between the cities B and C.



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